Problem 3: Show that $\mathcal{O}_{\mathbb{P}^n}(d)$ restricts to a projective line in $\mathbb{P}^n$ as a l.b. of degree d.

Proof: case $d \geq 0$: $PGL(n+1)$ acts transitively on lines in $\mathbb{P}^n$ and the line bundle $\mathcal{O}_{\mathbb{P}^n}(-1)$ is $GL(n)$ invariant (as a subbundle of the trivial bundle $\mathbb{P}(V) \times V$, $V \simeq \mathbb{C}^n$). Hence, we may assume that the line is $C = (Z_2 = Z_3 = \cdots = Z_n = 0)$.

Now Z_1^d is a global section of $\mathcal{O}_{\mathbb{P}^n}(d)$ (homog poly of deg d) and it does not vanish identically on C, so it restricts to a global section Z_1^d of $\mathcal{O}(d)|_C$.

Over the open chart $\overset{\mathbb{C} \simeq C \cap}{(z_0 \neq 0)}$ of C, Z_1^d corresponds, under the trivialization of $\mathcal{O}(d)|_{U_0}$, to the holo function $(Z_1/Z_0)^d$, which vanishes at $(1, 0, \dots, 0)$ to order d.

The section Z_1^d does not vanish on the chart $C \cap (Z_1 \neq 0)$ and so

$$\deg(\mathcal{O}_{\mathbb{P}^n}(d)|_C) = \deg(\text{divisor of } Z_1^d|_C) = d.$$

For $d \leq 0$ use the fact that $\deg : \operatorname{Pic}(\mathbb{P}^1) \to \mathbb{Z}$ is a homomorphism, so

$$\deg(\underbrace{\mathcal{O}_{\mathbb{P}^n}(-d)}_{(\mathcal{O}_{\mathbb{P}^n}(d))^*}|_{\mathbb{P}^1}) = -\deg(\mathcal{O}_{\mathbb{P}^n}(d)|_{\mathbb{P}^1})$$

$\square$