Hint for problem 3.3.6 page 143 in Huybrechts' book

In Section 2.1 page 57 the space of n-dimensional complex tori $\mathbb{C}^n/\Gamma$ was described by fixing a complex structure on the universal cover $\mathbb{C}^n$ and varying the lattice $\Gamma \subset \mathbb{C}^n$. For the current problem it is more convenient to fix the integral structure and vary the complex structure on the universal cover. So let V be a real 4-dimensional vector space with the fixed standard embedding of the full lattice $\Gamma := \mathbb{Z}^4$.

1. Show that the space of all constant complex structures on V (called almost complex structure in Definition 1.2.1 page 25) is naturally an open dense subset Q^0 of the Grassmannian $G(2, V_{\mathbb{C}})$ of 2-dimensional subspaces of the complexification $V_{\mathbb{C}} := V \otimes_{\mathbb{R}} \mathbb{C}$ of V. (Send I to $V_{\mathbb{C}}^{1,0}$).

2. Show that $G(2, V_{\mathbb{C}})$ is naturally isomorphic to the smooth hypersurface Q of degree 2 in the complex projective space $\mathbb{P}(\wedge^2 V_{\mathbb{C}}) \cong \mathbb{P}^5$ (the projectivization of the six dimensional second exterior power of $V_{\mathbb{C}}$)
$$Q := \{\alpha \in \wedge^2 V_{\mathbb{C}} \ : \ \alpha \wedge \alpha = 0\}.$$
Conclude that $G(2, V_{\mathbb{C}})$ is not contained in any hyperplane in $\mathbb{P}(\wedge^2 V_{\mathbb{C}})$.

3. Let λ be a non-zero integral element of $\wedge^2 \Gamma^* \subset \wedge^2 V_{\mathbb{R}}^* \subset \wedge^2 V_{\mathbb{C}}^*$ and let $I : V \to V$ be a complex structure. Let $\wedge^{p,q}(V, I)^*$ be as in Definition 1.2.7 page 27 with respect to the complex structure I induces on V^*. Show that λ belongs to $\wedge^{1,1}(V, I)^*$, if and only if $\lambda \wedge [\wedge^{2,0}(V, I)^*] = (0)$. Conclude that the subset of complex structures I for which λ belongs to $\wedge^{1,1}(V, I)^*$ is contained in a hyperplane of $G(2, V_{\mathbb{C}})$. Hence, the locus of complex structures I, for which
$$(V^*)^{1,1} \cap \wedge^2 \Gamma^* \neq (0)$$
is a countable union of hyperplane sections of Q^0.

4. Now translate the above linear algebra argument to the desired result for complex tori.