

Math 697B - Introduction to Riemann Surfaces - Fall 2004

MWF 10:10 → 1:00 LGRT 1234

Instructor: Eyal Markman

Office: LGRT 1242

Office Phone: 545-2788

E-mail: markman@math.umass.edu

Office hours: (Tentative!!!) Monday 11 am → 12:30 pm, Wednesday 2:30 → 4:00 pm, and by appointment.

Texts:

1. *Complex algebraic curves*, by Frances Kirwan, London Mathematical Society Student Texts, 23. Cambridge University Press, Cambridge, 1992.
2. *Lectures on Riemann Surfaces*, by Otto Forster, Springer-Verlag 1981

Kirwan's is an advanced undergraduate text, while Forster's is a graduate textbook, hence more advanced, precise, and technical. Many other excellent books are available. See the references in these two books, as well as W. Fulton's *Algebraic curves* (at the advanced undergraduate level) and Hartshorne's *algebraic Geometry* (definitely a graduate level textbook).

Course Plan: We will first cover the equivalent of chapters 2, 3, and some of 4, of Kirwan's book. The topic is complex affine and projective algebraic curves, with emphasis on plane curves. Highlights include: Bézout's theorem (on the degree of the intersection of two curves in the projective plane), the group law on a plane cubic curve, and the degree-genus formula for a smooth plane curve.

We will then proceed to the theory of compact Riemann surfaces and cover the equivalent of the first two chapters in Forster's book. A deep result states, roughly, that any compact Riemann surface is a (projective) algebraic curve. The study of Riemann surfaces uses complex analysis, while the study of algebraic curves uses algebra. We will develop the language of line bundles and sheaves on Riemann surfaces and their sheaf cohomology. Highlights include: elliptic functions, the Riemann-Roch and Serre's Duality Theorems, the Abel-Jacobi Theorem, and their many applications.

The material from Kirwan's geometric and intuitive book will serve as a gentle introduction motivating the more advanced techniques in Forster's book. The material in the first two chapters of Forster's book, if presented faithfully, takes a full semester. We will face some hard choices as we follow the above plan.

Homework: Will be assigned regularly. Students will present their solutions to homework problems in several specially scheduled evening problem sessions. Group work is encouraged, but individual papers should be handed in.

Grades: Will be determined by the homework and class participation.