# Stat 605: Problem set 5

1. Exercise 9.13. p. 62

2. Exercise 9.14. p.62

3. Exercise 9.15. p.62 (the density of the lognormal is on page 44).

4. Exercise 9.19, p.63 (this is a useful formula).
   *Hint:* Do the change of variable $s = x - t$ in $P(X > x) = \int_x^\infty f(t)dt$.

5. Exercise 9.20 and 9.21 on p. 63

6. Show that if $E[|X|^k] < \infty$ then for $0 < j < k$ $E[|X|^j] < \infty$.
   *Hint:* Divide the integral into the two regions $|X| \leq 1$ and $|X| > 1$.

7. Chebyshev and one-sided Chebyshev bounds

   (a) In class we have proved Markov and Chebyshev inequality for discrete random variables. Show that they hold for a random variable $X$ with an arbitrary law $P^X$. *Hint:* Do the same proof using general properties of the integral.

   (b) Let $a > b > 0$ and $0 < p < 1$, and let $X$ be the random variable with $P(X = a) = p$ and $P(X = -b) = 1 - p$. Apply Markov inequality to the random variable $(X + b)^2$ and conclude that if $Y$ is any random variable with $E[Y] = E[X]$ and var(Y) = var(X) then $P(Y \geq a) \leq p$ and that equality holds when $Y = X$.

   (c) Suppose that $E[Y] = 0$, $var(Y) = \sigma^2$, and $a > 0$. Show that $P(Y \geq a) \leq \sigma^2/(a^2 + \sigma^2)$, and there is a $Y$ for which equality holds. *Hint:* Use (b)

8. (a) Consider two random variable $X$ and $Y$ both in $L^1$. Show that the random variable $XY$ is not necessarily in $L^1$ (give a counterexample) but if $X$ and $Y$ are independent then $XY \in L^1$.

   (b) Find two random variables $X$ and $Y$ which are not independent but $E[XY] = E[X]E[Y]$