Stat 605: Problem set 3

1. Exercise 7.1, p.45

2. Suppose P is the distribution of some random variable X with distribution F_X and density $p_X(x)$

$$F_X(x) = \int_{-\infty}^{x} p_X(x)dx = P((-\infty, x]) = P(X \leq x)\,.$$

   (a) Compute the distribution function $F_{X^2}(x)$ of X^2 and then differentiate it to find the density 0f X^2.

   (b) Do the previous calculation of the normal distribution X. In that case X^2 is called the chi-square distribution.

3. Suppose X has a continuous density $p(x)$ and we have $P(\alpha \leq X \leq \beta) = 1$ and suppose $g : [\alpha, \beta]$ is strictly increasing and differentiable on (α, β). Show that $g(X)$ has the density

$$\frac{f(g^{-1}(y))}{g'(g^{-1}(y))} \quad \text{for } y \in (g(\alpha), g(\beta))\,.$$

   Hint: One option is to use the same strategy as in the previous problem.

4. Using the previous problem compute the density of log-normal distribution, i.e., the random variable $Y = e^X$ where X is a normal distribution.

5. Exercise 7.16, p.46

6. Let P be the probability measures equal to $P = \frac{1}{2}\delta_1 + \frac{1}{2}P_N$ where δ_0 is the probability measure with $P(\{1\}) = 1$ and P_N is law of an exponential random variable with parameter λ (i.e. $F(x) = 0$ for $x < 0$ and $F(x) = \int_0^\infty \lambda e^{-\lambda x}$ for $x > 0$). Compute the distribution function for P.

7. Exercise 7.17, p.46

8. Exercise 7.18, p.46