

Stat 605: Problem set 10

1. Exercise 18.1, p.164
2. Exercise 18.8, p.164
3. Exercise 18.9, p.164
4. Exercise 19.3, p.171
5. Exercise 20.3, p.178
6. Exercise 20.5, P.178
7. Exercise 20.8, p.179
8. Exercise 21.1, p.187
9. Exercise 21.8, p.187
10. (***Hit-or-miss method***)
 - (a) Suppose that you wish to estimate the volume of a set B contained in the Euclidean space \mathbf{R}^k . You know that B is a subset of A and you know the volume of A . The “hit-or-miss” method consists in choosing n independent points uniformly at random in A and use the fraction of points which lands in B to get an estimate of the volume of B . (We used this method to compute the number π in class.) Write down the estimate I_n obtained with this method and compute the variance $var(I_n)$. (This will be expressed in terms of the volume of A and B .)
 - (b) Suppose now that D is a subset of A and that we know the volume of D and the volume of $D \cap B$. You decide to estimate the volume of B by choosing n points at random from $A \setminus D$ and counting how many land in B . What is the corresponding estimator I'_n of the volume of B for this second method? Show that this second method is better than the first one in the sense that $var(I'_n) \leq var(I_n)$.
 - (c) How would you use this method concretely to improve the estimation of the number π ? Compute the corresponding variances.