Evolutionary game theory, Problem set 1

1. (Zeckhauser Paradox, *please do not try at home*). Imagine a standard game of Russian roulette where some bullets are loaded into a revolver with six chambers, the cylinder is then spun and the gun is pointed at your head. Would you be prepared to pay more to get one bullet removed if one bullet was loaded or if four bullets was loaded? It seems reasonnable to pay more if one bullet was loaded since, in this case, you are sure to live. Use the Von-Neumann Morgenstern theory of utility to show that in fact you should pay *more* if there are four bullets loaded.

   Hint: Consider the prizes D=Dead, A=Alive, L_X=alive after paying X, L_Y=alive after paying Y. Assign a utility to each prize and compare lotteries.

2. Many games are naturally not directly given in normal (or strategic) form, but rather in "extensive form" which consist of a series of successive actions. Think for example of playing poker or chess. A strategy for the normal form corresponds then to a succession of actions in the extensive form. To illustrate these concepts consider the following simple card game. First players I and II put \$1 in a pot. Next player I draws one card from a shuffled deck in which half of the card are black and half of the cards are red. Player looks at his card and then decides either to raise or fold. If player I folds he shows his card to player, the games ends, and player I gets one dollar if the card is red and player II gets \$1 is the card is black. If player I raises then he adds \$1 to the pot and player II can either meet or pass. If player II passes, then the game ends and player I takes the money in the pot. If player II meets he must add \$1 to the pot, player I shows the card to II and the game ends, if the card is red I takes the pot and if the card is black II gets the pot.

   (a) Express this game as a normal form game, and compute the corresponding payoff matrix. *Hint:* Player II has two choices and player I has four choices.

   (b) Determine the strictly dominated and weakly dominated strategies.

   (c) Compute the Nash equilibria.

3. Find all Nash equilibria for the following games given in bi-matrix form

   (a)

$$\begin{pmatrix} \boxed{\begin{array}{cc} & 1 \\ 2 & \end{array}} & \boxed{\begin{array}{cc} & 2 \\ 1 & \end{array}} \\ \\ \boxed{\begin{array}{cc} & 5 \\ 1 & \end{array}} & \boxed{\begin{array}{cc} & 1 \\ 2 & \end{array}} \end{pmatrix} \tag{1}$$

   (b)

$$\begin{pmatrix} \boxed{\begin{array}{cc} & 4 \\ 0 & \end{array}} & \boxed{\begin{array}{cc} & 6 \\ 5 & \end{array}} & \boxed{\begin{array}{cc} & 7 \\ 8 & \end{array}} \\ \\ \boxed{\begin{array}{cc} & 9 \\ 2 & \end{array}} & \boxed{\begin{array}{cc} & 5 \\ 6 & \end{array}} & \boxed{\begin{array}{cc} & 1 \\ 5 & \end{array}} \end{pmatrix} \tag{2}$$

4. Suppose a game $(\Gamma, S', \boldsymbol{\pi}')$ is derived from another game $(\Gamma, S, \boldsymbol{\pi})$ by eliminating some pure strategies which are weakly dominated. Show that if $\mathbf{p}'$ is a NE for $(\Gamma, S', \boldsymbol{\pi}')$ then it is also leads to a NE $\mathbf{p}$ for $(\Gamma, S, \boldsymbol{\pi})$. ($\mathbf{p}$ is obtained from $\mathbf{p}'$ by choosing the same probabilities $p_{j,\gamma} = p'_{j,\gamma}$ for the strategy $j \in S' \subset S$ and $p_{j,\gamma} = 0$ if $j \in S \setminus S'$.

5. Consider a two-players game with bi-matrix payoff

$$\begin{pmatrix} \boxed{\begin{array}{cc} & 1 \\ 5 & \end{array}} & \boxed{\begin{array}{cc} & 0 \\ 4 & \end{array}} \\ \\ \boxed{\begin{array}{cc} & 0 \\ 6 & \end{array}} & \boxed{\begin{array}{cc} & 1 \\ 3 & \end{array}} \\ \\ \boxed{\begin{array}{cc} & 4 \\ 6 & \end{array}} & \boxed{\begin{array}{cc} & 4 \\ 4 & \end{array}} \end{pmatrix} \tag{3}$$

   Which strategies are strictly dominated? weakly dominated? Discuss the iterative elimination of strategies for this game and show that different games occur depending on the order with which dominated strategies are eliminated. Find the Nash equilibria for this game.

6. Find the NE for the one-population (or symmetric) game with payoff matrix

$$\begin{pmatrix} 1 & 2+a & 0 \\ 0 & 1 & 2+a \\ 2+a & 0 & 1 \end{pmatrix} \tag{4}$$

7. Consider the following game with 3 players γ_1, γ_2 and γ_3 and strategies sets $S_{\gamma_1} = \{s_1, s_2\}$ $S_{\gamma_2} = \{t_1, t_2\}$, and $S_{\gamma_3} = \{u_1, u_2\}$. We give strategies with the two tables

u_1 (5)

	t_1	t_2
s_1	0 0 0	4 5 6
s_2	6 4 5	0 0 0

u_2 (6)

	t_1	t_2
s_1	5 6 4	0 0 0
s_2	0 0 0	0 0 0

The tables should be read as follows. If γ_3 plays u_1 use the first table and if γ_1 plays s_1 and γ_2 plays t_2 then the payoffs for γ_1 is 6, the payoff for γ_2 is 5 and the payoff for γ_3 is 4, etc.....

Find the NE for this game. *Hint:* Every player gets 0 payoff unless exactly one player uses his second strategy. The player using his second strategy get 5, the player before him gets 6, the player after him gets 4 with the cyclic ordering $\gamma_1 \to \gamma_2 \to \gamma_3 \to \gamma_1$.