

Math 623, Fall 2013: Problem set 7

- (a) Show that the Cauchy-Schwartz inequality for a general Hilbert space. *Hint:* Consider the quadratic equation in λ given by $(f + \lambda g, f + \lambda g)$.
 - (b) Show that we have equality in the Cauchy-Schwartz inequality if and only if $f = cg$ for some scalar c .
- Prove that directly from its definition that $l^2(\mathbf{N})$ is complete and separable.
- Exercise 5 on p. 194
- Exercise 7 on p. 194
- Exercise 11 on p. 195
- Consider the subspace S of $L^2([0, 1])$ spanned by the $f(x) = 1$, $f(x) = x$ and $f(x) = x^3$. Find an orthonormal basis of S and compute $P_S g$ where $g = x^2$ and P_S is the orthogonal projection on the subspace S .
- Exercise 18 on p. 197