Math 623, Fall 2013: Problem set 6

1. Suppose $f(x)$ on $\mathbf{R}^{d_1}$ is measurable and $g(y)$ on $\mathbf{R}^{d_2}$ is measurable. Show that $F(x,y) = f(x)g(y)$ on $\mathbf{R}^{d_1+d_2}$ is measurable.

2. Let f be an integrable function on $\mathbf{R}^d$. Show that the graph of f i.e., the set

$$\Gamma = \left\{(x,y) \in \mathbf{R}^d \times \mathbf{R} \,:\, y = f(x)\right\}$$

   is a measurable subset of $\mathbf{R}^{d+1}$ and that $m(\Gamma) = 0$.

3. Problem 18, p. 93

4. Problem 19, p.93.

5. (a) Consider the function $f(x,y) = ye^{-(1+x^2)y^2}$ if $x \geq 0$ and $y \geq 0$ and 0 otherwise. Integrate this function over $\mathbf{R} \times \mathbf{R}$ to show that $\int_0^\infty e^{-x^2}\,dx = \sqrt{\pi}/2$. Justify all your steps carefully!

   (b) Use part (a) to compute $\int_{\mathbf{R}} e^{-tx^2}\,dx = \sqrt{\pi/t}$ and to show that $\int x^{2n}e^{-x^2}\,dx = \frac{(2n)!\sqrt{\pi}}{4^n n!}$ (use Problem 1 in Hwk 5).