# Math 623: Homework 7

1. Prove the Cauchy-Schwarz inequality: for any $f, g$ in a Hilbert space $\mathcal{H}$ one has

$$|(f\,,\,g)| \leq \|f\|\,\|g\|\,,$$

and the equality holds if and only if $g = \alpha f$ for some $\alpha \in \mathbf{C}$. Show that it is true first if $f$ and $g$ are perpendicular or if $f$ and $g$ are collinear (i.e. $g = \alpha f$). For general $f$ and $g$ write $g = g_\perp + g_\parallel$ into a perpendicular part and a collinear part.

2. Exercise 4, p. 194
3. Exercise 5, p. 194
4. Exercise 6, p. 194
5. Exercise 7, p. 194
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