Lecture 1: Introduction and examples of games

We introduce here the basic objects involved in game theory. To specify a game ones gives

- The *players*.
- The set of all possible *strategies* for each player.
- The *payoffs*: if each player picks a certain strategy then each player receive a payoff represented by a number. The payoff for every player depends, in general, from the strategies of all other players. Payoffs have many different meanings, e.g., an amount of money, a number of years of happiness, the fitness in biology, etc... Our convention is that the highest payoff is deemed the most desirable one.

In the rest of this section we introduce some of the standard games in game theory, together with the "story behind them"

The prisoner's dilemma:. This is one of the most famous game in game theory. One possible story associated to it goes as follows. A prosecutor seeks two arrest two bank robbers. He lacks proofs of their involvement in the heist but has managed to have them arrested for a minor fraud charge. He offers to each them the following choices. If you confess but your accomplice does not then you will go free and your accomplice will be punished 8 years in jail. If you both confess then you will each get 6 years in jail. Finally, if none confess then they both will convicted for the minor fraud charge, say for 1 year in prison.

We will represent payoffs and strategies using a table. We will call the player Robert (R is for the "row" player) and Collin (C is for "column" player). For both Robert and Collin the strategies are confess or not confess.

		Collin	
		confess	not confess
Robert	confess	-6 -6	-8 0
	not confess	-6 -8	-1 -1

The rows represents the strategies of Robert, the columns the strategies of Collin. The numerical entries are understood as follows: in each box the lower left entry is the payoff for Robert while the the upper right entry is the payoff for Collin.

Solution of the prisoner's dilemma: To see what is the best option for the prisoner's dilemma one observes that if your accomplice confess then it is better for you to confess

(8 years in jails) rather than not confessing (10 years in jail). On the other hand if your accomplice does not confess it is better for you to confess (0 years in jails) rather than not confessing (1 years in jail). Therefore the outcome of the game is that rational players will both confess and end in jail for 8 years. We will see later that the pairs of strategies "confess" and "confess" is an equilibrium for this game. The "dilemma" here is that if both do not confess they would get both 1 year in jail instead of 8 years, a much more preferable outcome for both! However without communication between the players, there is no mechanism to enforce this.

Duopoly: A strategic situation similar to the prisoner's dilemma occurs in many different contexts. Imagine for example two countries who produce oil and each can choose between producing 2 millions barrels/day or 4 millions barrels/day. The total output will be then 4, 6, or 8 millions barrels/day and the corresponding price will be \$20, \$12, \$7 per barrel (a decreasing price reflects basic law of supply and demand). The payoff table is given by

		Country B	
		2 million	4 million
Country A	2 million	40 40	48 24
	4 million	24 48	28 28

Analyzing the game as in the prisoner's dilemma one finds that the best strategy is to produce \$4 million per day because it leads to a higher revenue no matter what the other country does. This is of course in many ways a socially bad outcome since it leads to wasting a precious resource and lower revenue for the oil producing countries.

Battle of the sexes: Robert and Chelsea are planning an event of entertainment. Above all they value spending time together but Robert likes to go to the game while Chelsea prefers to go to the ballet. They both need to decide what to do tonight without communicating with each other. To produce a numerical outcome we assume that the value 1 is given to having your favorite entertainment while a value 2 is assigned to being together. This leads to

		Chelsea	
		game	ballet
Robert	game	2 3	1 1
	ballet	0 0	3 2

It is useful to relabel the strategies as "selfish" which means game for Robert and ballet for Chelsea and "altruistic" which means ballet for Robert and game for Chelsea. Then the game has a more symmetric structure and we have the payoffs s

		Chelsea	
		selfish	altruistic
Robert	selfish	1 1	2 3
	altruistic	3 2	0 0

Finding what to do is here more complicated. If both players are altruisitic, which is consistent with wanting to be together, then the outcome is the worst possible. So in this game being a bit selfish is good. However if both players are purely selfish the outcome is not that great. The best possible outcome occur if one is selfish and the other is altruistic. This outcome is better for both than the other options. We will think then of the combination of strategies "selfish" and "altruistic" as a solution for this game. However there are 2 such outcomes, and so how should you decide who is selfish?

Chicken game: This game was famous during the cold war since it captures the essence of an arm race. But you may find yourself many other applications of this game. Imagine two drivers racing toward each other at high speed on a very narrow road. Each driver has the option to swerve or to race on. If one swerves while the other races on he is ridiculed and called a chicken. If both swerves it is a tie and if none swerve it ends in mutual destruction

A payoff table consistent with this game is

		Collin	
		swerve	race on
Robert	swerve	1 1	2 0
	race on	0 2	-10 -10

We leave it to the reader to interpret these numbers and to analyze strategic situation the game.

Matching pennies: This children is game is played as follows. You have a penny that you can show either as head or tail. If both pennies coincide then Robert wins and takes Collin's penny while if they do not Collin wins and takes Robert's penny. The payoffs are given by

		Collin	
		H	T
Robert	H	-1 1	1 -1
	T	1 -1	-1 1

Playing head or tail is not a winning strategy and it should be intuitively clear that the best option is is to play head or tail at random.

Rock-Paper-Scissor: This other well-known children game has three strategies and we have

Rock wins against scissor, scissor wins against paper, paper wins over rock

that is the strategies cyclically dominate each other. We shall encounter other situations where this structure occur but for now you may simply think of

Man eats chicken eats worm eats .....

A payoff matrix for the game is given by

		Collin		
		R	S	P
Robert	R	0 0	-1 1	1 -1
	S	1 -1	0 0	-1 1
	P	-1 1	1 -1	0 0

and every child will tell you that the best way to play is to pick a random strategy.

Conclusion: After all these examples we want to develop a number of tools to analyze a game in a more systematic manner. This will center around the idea of an "equilibrium" which is a pair of outcomes where both players are as happy as possible, or rather as little unhappy as possible.... We shall also see many applications of these games in many situations, some maybe rather unexpected like in evolutionary biology where the players (animals) are not necessarily rational beings.

Exercise 1: The snowdrift game: One possible story behind the game is the following: suppose two drivers are caught in a snowstorm and a big snowdrift blocks the road. To go home they have to clear the path. The fairest solution is for them to clear the path together. If one simply refuses to do it, the other driver may just do it because he wants to go home. But if both drivers have the same idea nobody goes home.

A variant of this game with numerical payoff is the following: I will give to Robert and Chelsea each a gift worth $40 if I receive $30 in cash. Their options are to either to pay the fee or not pay the fee knowing that if both of them decide to pay then then they will share the fee and pay $15 each. Write down a payoff table for the game.

Exercise 2: The ultimatum game: Consider the following experiment where \$100 is handed to Robert and he is given the task to split the amount of money between Robert and Chelsea any way he wants. Then Chelsea has the option to accept the deal and take the money offered, or to refuse in which case both go empty-handed. In most experiments Robert will propose a more or less fair deal, say 55-45 and Chelsea will accept. If Robert proposes a bad deal, like say 85-15 then often this deal will be rejected even though Chelsea would be better off accepting it and taking any money rather than having nothing.

Let us construct a simple game which captures the essence of this relation. Robert has two option, offer a fair split, say 60-40, or offer a unequal split, 85-15. Chelsea has also two options, accepting any offer or accepting only the fair offer. Write down the payoff matrix for the game.