Homework 9

1. Consider an assymetric version of the Cournot duopoly game where the marginal cost for firm 1 is c_1 and the marginal cost for firm 2 is c_2.

   (a) If $0 < c_i < \frac{1}{2}P_0$ what is the Nash equilibrium?

   (b) if $c_1 < c_2 < P_0$ but $2c_2 > P_0 + c_1$, what is the Nash equilibrium

2. Cournot model for m firms. Suppose m firms produce the quantities $Q_1, Q_2, \cdots, Q_m$ of the same product on the same market. The market price is given by $P(Q) = P_0(1 - Q/Q_0)$ with $Q = Q_1 + \cdots + Q_m$, c is the marginal cost of producing one item and the profit for firm i is given by

$$\Pi_i(Q_1, \cdots, Q_M) = Q_i[P(Q) - cQ_i] \, .$$

   (a) Generalizing the case of two firms, explain how to obtain the Nash equilibrium for this model.

   (b) Find the symmetric Nash equilibria, i.e. the Nash equilibria with $\widehat{Q}_1 = \cdots = \widehat{Q}_m$. *Hint: To avoid long computation assume from the start that the Nash equilibrium is symmetric. The best response equations will then reduce to solving one single equation rather than a system of m equations.*

   (c) Compute the price and profits in the symmetric Nash equilibrium. What happens for large m?.

3. Two adjacent countries (labelled by $i = 1, 2$) each have industries that emit pollution at a level e_i tons per year. Pollution from one country has a reduced effect on the other, so that the total level of pollution in country 1 is $E_1 = e_1 + ke_2$ (with $0 < k < 1$) and the total level of pollution in country 2 is $E_2 = e_2 + ke_1$. Initially, each country produces an amount of pollution e_0. However, the parliament in each country can vote to reduce the amount of pollution that it produces at a cost of c pounds per ton per year. The cost to the government-funded health service in each country increases with the total level of pollution as bE_i^2 . Construct the payoffs $\Pi_i(e_1, e_2)$ for each of the countries and determine the equilibrium level of pollution produced in each country, assuming that the parliaments vote simultaneously.

4. The Samaritan dilemma (also called the Welfare game): This dilemma occurs when deciding whether to provide help to someone and balancing whether the benefit provided is helpful or whether it is an incentive to being unproductive.

   Mr and Mrs Roberts have a very lazy child named Corey. They are willing to help Collin financially but they do not want to contribute to his distress by allowing him to loaf around so they announce they might help Collin if he does not find a job. Collin however seeks work only if he cannot depend on his parents for financial support and in addition, he may be unable to find work even he is searching for it.

(a) Explain why the payoff table captures all the aspects of this the strategic situation

		Collin	
		Seek work	Loaf around
Mr and Mrs Roberts	Help Son	2 3	3 -1
	Tough love	1 -1	0 0

(b) Compute all the Nash equilibria (pure and mixed) for this game. Draw the reaction curve diagram.

(c) A similar strategic situation occurs for example in welfare where providing aid is sometimes argued as offering a disincentive to work. Or similarly for foreign aid to poor country. Does the Nash equilibria shed some light on this dilemma?

5. Use reaction curves or the equality of payoffs theorem to compute all Nash equilibria, pure and mixed, for the following games:

   (a) The snowdrift game from Homework 8 #1

   (b) The Ultimatum game from Homework 8, #2

   (c) The Stag Hunt game with payoff matrix

		Collin	
		cooperate	alone
Robert	cooperate	4 4	3 0
	alone	0 3	3 3

   (d) The Battle of the sexes game with payoff

		Chelsea	
		game	ballet
Robert	game	2 3	1 1
	ballet	1 1	3 2

6. Compute the Nash equilibria for the generalized version of Rock-paper scissor given by

$$P_R = \begin{pmatrix} 0 & 2 & -1 \\ -1 & 0 & 2 \\ 2 & -1 & 0 \end{pmatrix}, \quad P_C = \begin{pmatrix} 0 & -1 & 2 \\ 2 & 0 & -1 \\ -1 & 2 & 0 \end{pmatrix}$$