

Homework 6

1. **Standard deviation and risk.** The *standard deviation* $\sigma(X)$ of a random variable is the square root of the variance that is $\sigma(X) = \sqrt{\text{Var}(X)}$. It characterizes the "spread" of the random variable X . If a random variable X has expected value μ and standard deviation σ , then X takes values which are on average at distance σ from μ .

Imagine you have the choice to invest in two stock funds: an american fund with a rate return X and an asian fund with rate of return Y . The rate of return X is a random variable with mean $\mu = .15$ and standard deviation $\sigma(X)$ and the rate of return Y is a random variable with mean $.15$ and standard deviation $\sigma(Y)$ with say $\sigma(Y) > \sigma(X)$. The larger the standard deviation, the riskier your investment is, since a large standard deviation makes it more likely that the rate of return is far from the μ . This can be both good or bad, since the rate of return could be either much larger than the average or much smaller.

Given the choice between these two investment show that the least risky investment is actually to diversify and invest money α of your fortune in the american stock fund and a proportion $1 - \alpha$ in the asian stock fund (assume that X and Y are independent). Determine for which proportion α you risk is minimal.

2. I give you a coin and make the claim that it is biased and that heads comes up with probability $.48\%$ of the times. You decide to flip the coin yourself a number of times to make sure that I was saying the truth. Use Chebyshev inequality to estimate how many times you should flip that coin to be sure that the coin is really biased with a 95% confidence. What about a 99% confidence?
3. (a) Consider the uniform random variable U on $1, 2, \dots, N$, with probability distribution $P(U = n) = \frac{1}{N}$ for $n = 1, 2, \dots, N$. Compute the mean and the variance of U .

Hint: You may use the equalities

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

and

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(Do you know how to prove these?)

- (b) Billy sells newspaper at the entrance of the metro. Every morning he buys 200 papers for \$1 each and sells them for \$1.50. He goes home after all his papers are sold or at noon. He can return unsold newspapers to the distributor for \$0.50 a piece. The demand for his newspaper is uniformly distributed between 151

and 250 and if the demand exceeds 200 then the demand is unfilled. Compute the expected value and the standard deviation of Billy's earning.

4. Three musicians pick seven pieces of music out of 22 pieces independently of each other. They play one piece of music together only if the piece is chosen by all three of them. Let X be the random variable which denote the number of pieces of music played. Determine the expected value and the variance of X and compute $P\{X \geq 1\}$.

Hint: To do this let X_i to be 1 if the three musicians picks the i^{th} music piece and 0 otherwise. Express X in terms of X_i . Are the X_i 's independent?

5. Show that the variance of the geometric random variable N with probability distribution $P\{N = n\} = (1 - p)^{n-1}p$ for $n = 1, 2, 3, \dots$ is $\text{Var}(N) = \frac{1-p}{p^2}$.
6. Suppose that you run the Monte-Carlo algorithm to compute π 10'000 times and observe 7932 points inside the circle. What is your estimation for the value of π ? Using Chebyshev describe how accurate your estimation of π , the answer should be in the form should be in the form: based on my computation the number π belong to the interval $[a, b]$ with probability .95.