Homework 4

1. Solve the following difference equations

   (a) $2x(n) - 5x(n-1) = 0$, $x(0) = 2$.

   (b) $2x(n) - 5x(n-1) = 3$, $x(0) = 3$.

   (c) $2x(n) - 5x(n-1) = 3n$, $x(0) = 0$.

   (d) $2x(n+1) - 7x(n) + 3x(n-1) = 0$, $x(0) = 1, x(1) = 2$.

   (e) $2x(n+1) - 7x(n) + 3x(n-1) = 2 + 2^n$, $x(0) = 3, x(1) = 0$.

2. Your mortgage is for 30 years with a fixed annual rate of 4% compounded monthly.

   (a) If you borrow \$150,000 today, what is the total amount of money will you pay back to the bank during the next 30 years?

   (b) You can afford a down-payment of \$15,000 and a monthly payment of no more than \$1250. What is the value of the most expensive house can you buy?

3. Your retirement account guarantees a fixed rate of 8% per year *paid yearly*. You start saving for retirement at age 30 with a target retirement age of 65 and \$0 in your saving account. Use first order difference equations to answer the following questions (all interests and payments are computed on a yearly basis).

   (a) You set aside \$500 every month (or \$6000 a year). How much money will you have for your retirement?

   (b) You want to retire with \$500,000. How much should you save every year?

   (c) You assume that your salary will increase 5% every year and so you decide your contribution should also increase by 5% every year. If your starting contribution is \$500 every month how much money will you have saved at retirement age?

   (d) Assuming again that your contribution is increasing by 5% every year, what should your starting contribution be if if you want to reach \$500,000 by retirement age?

4. In the powerball with with a jackpot of \$ 40 millions, you can either receive a lump sum of \$ 27 million today or to receive \$ $1\frac{1}{3}$ million per year for the next 30 years. To compare the two options assume that you can get guaranteed interest rate of α percent yearly (compounded annually) and assume you invest and save all your money for 30 years.

   (a) Write down two difference equations for each of the two options ($x(n)$ is the value of investment after n years in million dollars).

   (b) For which interest rate α is the option of a lump sum better?