Homework 3

1. Suppose you have three boxes. One boxes contains two gold coins, the second box contains two silver coins and the third box contains one gold coin and one silver coin. Pick a box at random, pick a coin at random in the box. Suppose that the coin you picked is a gold coin, what is the probability that the other coin in the box you picked is also a gold coin?

2. Suppose Math 478 has two section. In section I there are 12 female and 18 male students. In section II there are 20 female and 15 male students. The professor picks a section at random and then picks a student at random in that section. Compute

   (a) Probability that the student chosen is a female.

   (b) The conditional probability that the student is in section II given that she is a female

3. Consider a deck consisting of the thirteen diamonds cards which we order from ace (the lowest) to king (the highest). Three players are each given one of these 13 cards.

   (a) Suppose the first player has the card x. Compute the probability that he has the highest card among the three players for all values of x.

   (b) Suppose the second player has the card y. She now asks the first player whether she thinks that she has the highest card among the three. The first player, being a math major, will say YES if the probability that she has the highest card exceeds $\frac{1}{2}$ and NO if it is below $\frac{1}{2}$. If the first player says YES, what is the probability that the second player has the highest card among the three players for all values of y.

   (c) Suppose the third player has the card z. The third player which heard the answer of the first player to the second player now asks the second player if she thinks she had the highest card among the three. If the second player says YES compute the probability that the third player has the highest card among the three players for all values of z.

4. **False positives, Sensitivity and Specificity.** Bayes formula is used routinely in epidemiology. Suppose we deal with a disease and we have test for the disease. We know

   - The *sensitivity* of the test is 99%, this means that if you have the disease, the test is positive with probability 0.99.

   - The *specificity* of the test is 98%, this means that if you do not have the disease, the test is negative with probability 0.98.

- The *prevalence* of the disease is one in two hundred, this means that the probability to carry the disease is 0.005.

The probability that someone with a positive test actually has the disease is called the *positive predictive value* of the test. The probability that someone with a negative test actually does not have the disease is called the *negative predictive value* of the test.

Express the positive and negative predictive value of the test using conditional probabilities and compute them using Bayes formula.

5. **Monty's Hall problem.** In this problem you will analyze how important the assumptions are in Monty's Hall problem.

    (a) Assume that the prize is randomly put behind one the three doors. However after watching the game many times you notice that the show host who is usually standing near door 1 is lazy and he tends to open the door closest to him. For example if you pick door 1, he opens door 2 75% of the time and door 3 only 25% of the time. Does this modify your probability of winning if you switch (or not switch)?

    (b) Suppose you have been watching Monty's Hall game for a very long time and have observed that the prize is behind door 1 45% of the time, behind door 2 40% of the time and behind door 3 15% of the time. The rest is as before and you assume that the host opens a door at random (if he has a choice).

        i. When playing on the show you pick door 1 again and the host opens one empty door. Should you switch or not? Compute the various cases.

        ii. If you know you are going to be offered to switch would it be better to pick another door than door 1? Justify your choice by computing the corresponding probabilities.