Homework 1

1. What odds should you give in favor of the following event?

   (a) A card chosen at random from a 52-card deck is an ace.

   (b) Exactly two heads will turn up when three coins are tossed.

2. Bob and Maria are taking a math class with final grades A, B or C.

   The probability that Bob gets a B is .3 and the probability that Maria gets a B is .4. The probability that neither gets an A but at least one gets a B is .1. What is the probability that at least one gets a B but neither gets a C?

   Hint: Write down the sample space S and consider the events $E = \{Bob\ gets\ a\ B\}$ and $F = \{Maria\ gets\ a\ B\}$. What are $E \cap B$ and $E \cup B$?

3. In a soccer tournament (with many teams) the odds that Arsenal wins are given as 2 to 7 while the odds that Liverpool wins are given as 1 to 5. Give the odds that either Arsenal or Liverpool wins?

4. The German Lotto consists of choosing 6 different numbers out of $1, 2, \cdots, 49$. On June 21 in 1995, the drawing produced the numbers 15-25-27-30-42-48. Quite remarkably these were exactly the same numbers that on December 19 in 1986 and the first in the 3016 drawings of the german lotto that the same sequence had been drawn twice. What an amazing coincidence! Or is it really? Compute the probability that the same sequence of 6 numbers does not appear twice in 3016 drawings.

5. As seen in class if there are 23 people in a room the probability of having two people with the same birthday is more than $1/2$. In our class of 36 people what is this probability? For comparison compute the probability that somebody has the same birthday than you in our class? Are you surprised by the result?

6. In a horse race there are four horses and the amount of money which has been betted on each horse is

Euler	525
Bernoulli	900
Laplace	225
Fourier	350

   (a) According to the amount bet by all the players estimate the "true" odds that each horse will win.

   (b) What are the betting odds given to you by the betting house (assuming they take a %15 cut).

7. (a) Suppose you roll three dice repeatedly say m times. Compute the probability that he three dice will turn up sixes *at least once.*

   (b) How many times should you roll the three dice so that the probability you see three sixes at least once exceeds $\frac{1}{2}$?

8. The (moneyline) odds of the NHL game Columbus against Boston playing on January 23 2016 are given on www.oddsshark.com as Columbus (+175) and Boston (-210).

   (a) What are the corresponding fractional odds and numerical odds.

   (b) Alice happens to like gambling but at the same is a bit risk averse (not a great combination...). She decide to split her money (say \$ 100) and bet on both teams at the same time. She will bet \$$x$ on Columbus and \$$(100 - x)$ on Boston. For a given x what is the *minimal* amount of money, $M(x)$, she will receive, no matter what happens.

   (c) Find the optimal choice x^* which will maximize $M(x)$.

   (d) Comment on the choice x^* obtained in (c)? What does it say about the commission that the betting house takes on bets?