MATH. 421 – EXAM #2

11/22/99 **NAME:**__________________________

1) (20 points) Compute the integral $\int_C (y + ix^2)dz$, where C is the triangle with vertices at the points 0, 1, and $1 + i$ (traversed counter- clockwise).

2) (20 points) Determine whether the following statements are true or false. **Justify your answers.**

a) If C denotes the unit circle traversed counter-clockwise, then $\displaystyle\int_C \frac{dz}{e^{iz}} = 2\pi i$.

b) If $f(z)$ is an entire function, C is the unit circle traversed counter-clockwise, and $|z_0| < 1$, then

$$\int_C \frac{f'(z)}{z - z_0}\, dz \;=\; \int_C \frac{f(z)}{(z - z_0)^2}\, dz$$

c) Let α and β be arbitrary complex numbers and let C be a path from α to β. Then, $\displaystyle\int_C \bar{z} dz \; = (\bar{\beta}^2 - \bar{\alpha}^2)/2.$

3) (15 points) Let C be the square with vertices at the points $\pm 3 \pm 3i$ (oriented counter-clockwise). Compute $\displaystyle\int_C \frac{z^2\,dz}{(z-i)^2(z+1)}$

4) (10 points) Compute $\displaystyle\int_C \frac{dz}{z+1}$,

where C is the path indicated by the picture.

5) (7 points) Let $\displaystyle\sum_{n=0}^{\infty} a_n\, z^n$ be the Taylor expansion around $z_0 = 0$ of the function $f(z) = \dfrac{e^z}{1-z^3}$. Find the coefficient a_3.

6) (20 points) Find the Taylor expansion of the following functions at the indicated points: **a)** $f(z) = \dfrac{1}{z(z+1)}$ around $z_0 = 1$.

b) $f(z) = \mathrm{Log}(z+1)$ around $z_0 = 0$.

7) (8 points) Prove that $\left| \int_C \frac{z^2}{1+z}\, dz \right| < 12\pi$, where C is the piece of the circle $|z| = 4$ going from 4 to $4i$ counter-clockwise.