DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
FINAL EXAM: MATH 131 Spring 2003
20 May 2003

Your Name: _____

Your Instructor's Name: _____

This exam paper consists of 9 questions. The value of each question is as indicated. It has 8 pages, including this one.
On this exam, you may use a calculator, but no books or notes.
It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

This space reserved for marking the exam.

1. (15) _____

2. (10) _____

3. (10) _____

4. (10) _____

5. (10) _____

6. (10) _____

7. (10) _____

8. (10) _____

9. (15) _____

TOTAL (100)

(1) [15] Let $f(x) = 3x^5 - 20x^3 + 3$.
   (a) For which numbers x is this function increasing? For which is it decreasing? Express your answers in terms of intervals.
   (b) For which intervals is this function concave up? concave down?
   (c) Find the absolute maxima and minima of $f(x)$ for x in the interval $[-1, 3]$.

(2) [10] Suppose that we have a function $y(x)$ defined implicitly by the equation

$$y^3(2-y) \;=\; x^2.$$

(a) Find an expression for $\dfrac{dy}{dx}$.

(b) Find the equation for the tangent line to this curve at the point $(x, y) = (-1, 1)$.

(3) [10] Using the definition of the derivative as a limit of a quotient, compute the derivative of the function

$$f(x) \;=\; x^2 - 2x + 2$$

at the point $x = 2$.

(4) [10] At noon a ship A is 35 miles due north of a ship B. The ship A is traveling south at a speed of 14 miles per hour, and the ship B is traveling east with a speed of 20 miles per hour. Find a general expression for the distance between these two ships at any time t (measured in hours since noon). How fast is this distance increasing at 1:00 PM?

(5) [10] Compute $\lim_{x \to 0} \dfrac{1 - \cos(x)}{3x^2}$.

(6) [10] Suppose that we define a function $f(x)$ as follows.

$$f(x) = \begin{cases} x\ln(x^2) & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$$

Determine the value of the constant k so that $f(x)$ is continuous at $x = 0$. Show all of your work.

(7) [10]

(a) Let $y = 3x^2 - 2$ and $z = \sin(y)$. Note that with these definitions, z is a function of x. Compute $\dfrac{dz}{dx}$.

(b) Compute the derivative of the following function.

$$\sqrt[3]{\sin(x^3)} \cdot \ln(\tan(x))$$

(8) [10] Use logarithmic differentiation to compute y', where

$$y = \left(\frac{x^2-1}{x^2+1}\right)^x .$$

(9) [15] Find the dimensions and area of the largest rectangle with base along the x-axis that is contained in the region bounded by the x-axis and the curve $y = 10 - x^4$.