DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131 FALL 2002
FINAL EXAM

1. Consider the polynomial function $f(x) = \frac{x^4}{4} + \frac{x^3}{3} - x^2 + 2$

   a) [8] Determine analytically the intervals where the function is increasing and those where it is decreasing. Find also all local maxima and local minima. Show all your analytical steps.

   b) [8] Determine analytically the intervals where the function is concave upward and those where it is concave downward. Find also all points of inflection. Show all your analytical steps.

   c) [4] Support your results in parts a) and b) by a graph of the function $f(x)$ in a view window of appropriate size. Indicate clearly the inflection points and the local maxima and minima.

2. [10] Consider the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1.$

   Use implicit differentiation to show, that the tangent line to the ellipse at the point $(x_0, y_0) = (2, \sqrt{2})$ has the equation $x + \sqrt{2}y = 4$.

3. [15] Compute the following limits, using analytical steps (limit laws and/or L'Hospital's Rule). Justify all your steps!

   a) $\lim_{x\to\infty} \frac{1}{x} \ln\left(\frac{1}{x}\right)$

   b) $\lim_{x\to 2} \frac{x^2 + x - 6}{\cos(x-2)}$

   c) $\lim_{x\to 0} (1+x)^{\frac{1}{\sin(x)}}$

4. [15] A rectangular banner has a red border and a white center. The width of the border at the top and at the bottom is 1 foot and along the sides it is $\frac{1}{2}$ foot. The total area is 32 square feet. Find the dimensions of the banner, which maximize the area of the white center. Justify your answer and show all your work.

5. [10] Find all the horizontal and vertical asymptotes of the function

$$f(x) = \frac{x^3 + 4x + 6}{2x^3 - 8x^2}$$

   Show all the analytical steps involved.

6. [15] A radar station on the ground is tracking an aircraft, which is flying horizontally at an altitude of 3 miles. The radar signal indicates, that the distance between the aircraft and the radar station is 5 miles and the distance is increasing at a rate of 800 miles per hour. Calculate the speed of the aircraft. Show all your work!

7. [15] a) Find the global maximum and the global minimum of $f(x) = xe^{-x}$ on the interval $0 \le x \le 2$.

   b) Sketch the function over the interval, marking clearly the global maximum and the global minimum.