DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131 Fall 2003
EXAM #2

Your Section Number: _____

Your Instructor's Name: _____

Print Your Name: _____

Sign Your Name: _____

This exam consists of 7 questions. It has 8 numbered pages, where the last is a blank page for scratchwork.

On this exam, you may use a calculator and a page of your own notes, but no books.

It is not sufficient to just write the answers. You must show how you arrive at your answers unless instructed otherwise. If you draw a graph, show the numerical scale on each axis.

Leave the space below empty!

1. (10) _____
2. (15) _____
3. (15) _____
4. (15) _____
5. (15) _____
6. (15) _____
7. (15) _____

TOTAL (100)

1. (10 points) Let $f(x) = e^x + e^{-x}$. Determine $f^{(99)}(x)$ and use it to evaluate $f^{(99)}(5)$.

2. Consider the ellipse $4x^2 + y^2 = 16$.

   a) (10 points) Use implicit differentiation to compute the slope of the tangent line to the ellipse at the point $(\sqrt{3}, 2)$, showing steps.

   b) (5 points) Check your result by sketching the graph (with scale marked on each axis) of $y = \sqrt{16 - 4x^2}$.

3. a) (10 points) Find the derivative $\dfrac{dy}{dx}$ of the function y, implicitly defined by $x^2 + y^3 = 8$.

b) (5 points) Find all points on the curve $x^2 + y^3 = 8$ where the tangent line is horizontal: $dy/dx = 0$.

4. Differentiate each function, but *do not use logarithmic differentiation* and *do not simplify results*:

   a) (7 points) $y = \ln(\ln(\ln x))$

   b) (8 points) $y = (x^2 + 3)^{10} \tan(x^2)$

5. Let $y = x^2\, e^x$.

   a) (7 points) Use the **product rule** to compute dy/dx.

   b) (8 points) Use **logarithmic differentiation** to compute dy/dx.

6. (15 points) An observer, at a fixed distance of 300 meters from the launch pad of a rocket, watches it ascend vertically at 60 meters per second. Find the rate of change of the distance between the rocket and the observer when the rocket is 400 meters high. **Show your steps!**

7. a) (7 points) Find the linearization $L(x)$ of the function $f(x) = e^x$ at the point $a = 0$.

b) (5 points) Use the linearization to approximate $e^{0.01}$.

c) (3 points) Is this approximation greater than or less than the value produced by a calculator? **Justify your answer.**