DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131 Fall 2002
EXAM 1

Your Name: _____

Your Instructor's Name: _____

This exam paper consists of 7 questions. It has 9 pages, where the last is a blank page.

On this exam, you may use a calculator, but no books or notes.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

If you draw a graph, you must include the value of the range variables and show the tick marks on the axes, if any.

If your drawing includes the graphs of more than one function, you must label them so that we can tell which is which.

1. (15) _____

2. (15) _____

3. (15) _____

4. (10) _____

5. (15) _____

6. (15) _____

7. (15) _____

TOTAL (100)

1. a) (10) Evaluate the limit

$$\lim_{x \to 2} \frac{2x^2 - 5x + 2}{x^3 - 4x}$$

by an algebraic procedure. Show all your algebraic steps.

b) (5) Support your result in part a) with a calculator. Describe your procedure and show as much data as possible.

2. Given the function
$$f(x) = \begin{cases} x^2 + 1 & \text{for} \quad x \le 1 \\ ax + b & \text{for} \quad x > 1, \end{cases}$$

where a, b are constants.

a) (5) Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

b) (5) What conditions do a and b have to satisfy, so that $f(x)$ is continuous at $x = 1$.

c) (5) Find a and b, such that $f(x)$ is everywhere differentiable.

3. a) (10) Find all horizontal and vertical asymptotes for the function

$$f(x) \ = \ \frac{3x^2 + 1}{(2x + 3)(x - 1)}$$

Explain your reasons.

b) (5) Graph the function in part a) indicating all horizontal and vertical asymptotes.

4. (10) Determine the derivative $f'(x)$ of the given function **using the definition of the derivative as a limit**

$$f(x) = \frac{x+1}{x-1}$$

5. A ball is dropped from a tower of height H feet. Its height after t seconds, in feet above the ground, is given by the formula

$$h(t) \;=\; H - 16t^2.$$

a) (5) Find the velocity of the ball after t seconds.

b) (5) It is observed, that the ball hits the ground with a speed of 128 feet per second. How long was the ball in the air? Hint: The speed is the absolute value of the velocity.

c) (5) Use the information in part b) to determine the height of the tower.

6. a) (8) Suppose a is a number, such that

$$\lim_{h \to 0} \frac{a^h - 1}{h} = 17.$$

Find an equation for the tangent line to the graph of $y = a^x$ at the point $(0, 1)$.

b) (7) Find a formula for the derivative $f'(x)$ of the following function

$$f(x) = x^{1984} + \frac{1}{x\sqrt{x}} - 2002e^x.$$

7. a) (10) Find a formula for the derivative $f'(x)$ of the function

$$f(x) = \frac{x}{x-5}.$$

b) (5) Find the equation of the tangent line to the graph of $y = f(x)$ at the point $(4, -4)$.