

Math 331, Spring 2018: Midterm practice problems

1. Solve the initial value problem $\frac{dy}{dt} = y^2 [t + \cos(t)]$ with $y(0) = 6$.
2. Solve the initial value problem $y' = \frac{3x \sin(x^2) - 1}{3 + 2y}$ with $y(0) = 2$.
3. Mary initially deposits \$1000 in a savings account that pays interest at the rate of 5% per year (compounded continuously). She also arranges for \$25 per week to be deposited automatically into her account.
 - (a) Assume that weekly deposits can be approximated by continuous deposits. Write down an initial value problem for her account balance $S(t)$ over time (t measured in years).
 - (b) How long does she needs to save to buy a \$5000 car?
4. The half-life of a radioactive substance is 2 days. Find the time required for a given amount of the material to decay to 1/10 of its original mass.
5. A radioactive material loses 25% of its mass in 10 minutes. What is its half-life?
6. At what yearly rate of interest, compounded continuously, will a bank deposit double in value in 8 years?
7. Newtons law of cooling states that if an object with temperature $T(t)$ at time t is in a medium with temperature T_m the rate of change of T at time t is proportional to $T(t) - T_m$, thus T satisfies a differential equation of the form

$$T' = -k(T - T_m)$$

Here $k > 0$, since the temperature of the object must decrease if $T > T_m$, or increase if $T < T_m$. Well call k the temperature decay constant of the medium.

- (a) A thermometer is moved from a room where the temperature is 70F to a freezer where the tem- perature is 12F . After 30 seconds the thermometer reads 40F. What does it read after 2 minutes?
 - (b) An object is placed in a room where the temperature is 20C. The temperature of the object drops by 5C in 4 minutes and by 7C in 8 minutes. What was the temperature of the object when it was initially placed in the room?
8. Consider the following equation for a certain population of squirrels given by $P(t)$ (t is measured in years).

$$\frac{dP}{dt} = 2P \left(1 - \frac{P}{2}\right) (P - 1)$$

- (a) Find all the equilibrium points of the equations. Draw the phase line and determine the stability of each equilibrium points.

- (b) Make a graph of the solutions with initial conditions $P(0) = 1/4$, $P(0) = 3/2$, and $P(0) = 3$.
- (c) At a certain time the hunting of squirrels become permitted and the law allows that a certain percentage α of the squirrel population be eliminated every year. A new equation for the squirrel population is then

$$\frac{dP}{dt} = 2P \left(1 - \frac{P}{2}\right) (P - 1) - \alpha P$$

The IALS (International Association for the Liberation of Squirrels) asserts than no more than 10% of squirrels should be eliminated every year (i.e $\alpha = 0.1$), otherwise the population would go extinct. On the contrary the UHA (United Hunters of America) asserts that it is safe to hunt half of the squirrel population every year (i.e. $\alpha = 0.5$). Analyze the systems as α varies and determine who is right from the IALS or the UHA.

9. Solve the initial value problem $\frac{dy}{dt} = -3y/t - 2 - t^{-4}$, $y(1) = 4$.
10. Solve the initial value problem $\frac{dy}{dt} = -e^t/y$, $y(0) = -2$.
11. A home buyer can afford to spend no more than \$1000 per month on mortgage payments. Suppose that the interest rate is 5% (per year) and that the term of the mortgage is 20 years. Assume that interest is compounded continuously and that payments are also made continuously.
- (a) Determine the maximum amount that this buyer can afford to borrow.
- (b) Determine the total interest paid during the term of the mortgage
12. Solve the initial value problem $\frac{dy}{dt} = yt + 2t$, $y(3) = 2$.
13. Solve the initial value problem $\frac{dy}{dt} = 9y + e^{-3t}$, $y(0) = 3$.
14. Solve the initial value problem $\frac{dy}{dt} = 6y + 2t - 4$, $y(0) = 3$.
15. Solve the initial value problem $\frac{dy}{dt} = \frac{1 + y^2}{ye^{-x}}$, $y(0) = -2$.
16. Solve the initial value problem $\frac{dy}{dx} = 1 + y^2$, $y(0) = -2$.
17. Solve the initial value problem $\frac{dy}{dx} = y(y + 2)$, $y(0) = 1$.
18. Find the general solution of $x^2 + y^2 + 2xyy' = 0$.

19. Solve the initial value problem $(\sin(x) - y \sin(x) - 2 \cos x) + \cos(x)y' = 0$, $y(0) = -1$.
20. Find the general solution of $xy^2 + 2xyy' = 0$.
21. Find the general solution of $y' = \frac{y}{x} + e^{-y/x}$ *Hint: This one calls for a change of variable.*
22. Find the general solution of $\frac{dy}{dx} - y = xy^2$. *Hint: This one calls for a change of variable.*
23. Find all functions $M(x, y)$ such that the equation $M(x, y)dx + (x^2 - y^2)dy = 0$ is exact.
24. A tank initially contains a solution of 10 pounds of salt in 60 gallons of water. Water with $1/2$ pound of salt per gallon is added to the tank at 6 gal/min, and the resulting solution leaves at the same rate. Find the quantity $Q(t)$ of salt in the tank at time $t > 0$.
25. Consider the differential equation $\frac{dy}{dt} = 3y^3 - 12y$
- Find the equilibrium points, draw the phase line, and identify the stability of the equilibrium points.
 - Sketch the solutions with initial conditions $y(0) = 2$, $y(0) = -1$.