UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 15th, 2022**
1:00-3:00 pm

Your Name (Last, First)________________________________

Student ID Number__________________

Signature________________________ **Section Number** _____

Section	Instructor	Class Time		Section	Instructor	Class Time
1	Manas Bhatnagar	MWF 12:20-1:10pm		11	Sean Hart	MW 4:00-5:15pm
3	Vefa Goksel	MWF 11:15-12:05pm		12	Garyfallia Katsimiga	TuTh 10:00-11:15am
5	Manas Bhatnagar	MWF 1:25-2:15pm		13	Garyfallia Katsimiga	TuTh 8:30-9:45am
6	Catherine Benincasa	MW 2:30-3:45pm		14	Carolyn Broz	TuTh 2:30-3:45pm
7	Jinguo Lian	MWF 9:05-9:55am		16	Sean Hart	MW 2:30-3:45pm
8	Jinguo Lian	MWF 10:10-11:00am		17	Richard Buckman	MWF 9:05-9:55am
9	Richard Buckman	MWF 10:10-11:00am		18	Aubain Nzokem	TuTh 2:30-3:45pm
10	Kevin Sackel	TuTh 1:00-2:15pm		19	Kevin Sackel	TuThu 8:30-9:45am
				20	Carolyn Broz	TuTh 4:00-5:15pm

- Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.
- The above applies until you have submitted your exam to us. Do not use a cell phone or talk while waiting in line, and please wait until you exit the building to discuss anything, both for the benefit of others still taking the exam, and to prevent unintentionally spoiling the exam.
- There are six (6) questions and 15 pages. Please check if you have consecutive page number from 1 to 15, if not, please raise your hands let proctors know. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (**Answers given without proper justification may receive no credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	18	
3	16	
4	16	
5	16	
6	16	
Free	2	
TOTAL	100	

#1. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(1a) (8 points) Let $f(x) = x^2 + 2$. Use the limit definition of the derivative to find $f'(x)$. (e.g. Do not use the power rule, etc.)

(1b) (8 points) Given $f(x) = (\cos(x))^x$, find $\dfrac{df}{dx}$ using any differentiation methods.

#2. (18 points) Evaluate the following limits:

(2a) (6 points) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x^2}$.

(2b) (6 points) $\lim_{x \to \infty} x \sin\left(\frac{3\pi}{x}\right)$.

(2c) (6 points) $\lim_{x \to 0^+} x^x$.

#3. (16 points) Let $f(x) = 28 + 15x + 6x^2 - x^3$. We know that f is defined for all real numbers and that: $f'(x) = 15 + 12x - 3x^2$, $f''(x) = 12 - 6x$.

(3a) (8 points) Find the interval(s) on which f is increasing. Find the interval(s) on which f is decreasing. Determine the x-coordinates of all local maxima and local minima.

(3b) (8 points) Using the same function, find the interval(s) where the function is concave up. Find the interval(s) where the function is concave down. Determine the x-coordinates of all points of inflection of the graph of f(x).

(For convenience, $f(x) = 28 + 15x + 6x^2 - x^3$, with $f'(x) = 15 + 12x - 3x^2$, and $f''(x) = 12 - 6x$.)

#4. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(4a) (8 points) A rectangle has one of its sides on the x-axis and corners of the opposite side on the piece of the parabola $y = 36 - x^2$ above the x-axis. Find the largest possible area of such a rectangle.

(4b) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = \dfrac{x^3}{3} - x$ on the interval $[0, 3]$.

#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(5a) (8 points) A particle is moving with the velocity given by: $a(t) = \cos(t) + t$. Given that $v(0) = 0$ and $s(0) = 0$, find the position of the particle, $s(t)$.

(5b) (8 points) Since the function $f(x) = \sqrt{x}$ is continuous on the interval $[0, 1]$, and differentiable on the interval $(0, 1)$, the Mean Value Theorem applies, showing the existence of c in the interval $(0, 1)$ with certain properties. State the equation which holds and find a value of c demonstrating it.

#6. (16 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(6a) (8 points) Given a function $f(x) = 1 - x$. Write a definite integral representing the exact area of the region under the curve $y = f(x)$ on the interval [0, 1]. Evaluate this integral using the definition of the definite integral as a limit of Riemann sums. Do not use the fundamental theorem of calculus here. You can use the following identities for sums of powers of consecutive positive integers.

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

(6b) (8 points) Evaluate the Riemann sum for $f(x) = 1 - x^2$, $-1 \le x \le 1$ with four subintervals, taking the sample points to be the midpoints.

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