UNIVERSITY OF MASSACHUSETTS AMHERST DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131		Exam 2		November 7:00-9:0	9, 2022 00 p.m.
Your Na	me (Last, First)				
Student 3	ID Number				
Signatur	е	Section Number			
Section	Instructor	Class Time	Section	Instructor	Class Tin

Section	Instructor	Class Time	Section	Instructor	Class Time
1	Manas Bhatnagar	MWF 12:20-1:10pm	11	Sean Hart	MW 4:00-5:15pm
3	Vefa Goksel	MWF 11:15-12:05pm	12	Garyfallia Katsimiga	TuTh 10:00-11:15am
5	Manas Bhatnagar	MWF 1:25-2:15pm	13	Garyfallia Katsimiga	TuTh 8:30-9:45am
6	Catherine Benincasa	MW 2:30-3:45pm	14	Carolyn Broz	TuTh 2:30-3:45pm
7	Jinguo Lian	MWF 9:05-9:55am	16	Sean Hart	MW 2:30-3:45pm
8	Jinguo Lian	MWF 10:10-11:00am	17	Richard Buckman	MWF 9:05-9:55am
9	Richard Buckman	MWF 10:10-11:00am	18	Aubain Nzokem	TuTh 2:30-3:45pm
10	Kevin Sackel	TuTh 1:00-2:15pm	19	Kevin Sackel	TuThu 8:30-9:45am
			20	Carolyn Broz	TuTh 4:00-5:15pm

• Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.

- There are six (6) questions and 14 pages. Please check that you have all pages, that they are consecutive, and that there are no duplicates. Otherwise, please raise your hand and let proctors know. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (Answers given without proper justification may receive no credit.)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	16	
4	18	
5	16	
6	16	
Free	2	
TOTAL	100	

#1. (16 points) Find the derivatives of the following functions. You do NOT need to simplify your answer. BTW, these two problems are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to solve part (b).

(1a) (8 points)
$$f(x) = \pi^x x^{100} + \frac{1 - x^2}{1 + x^3} + \log_3(\cot^2(x)) + \arcsin(2x).$$

(1b) (8 points) Find the derivative of g(x). You do NOT need to simplify.

$$g(x) = x^3 \sqrt[3]{x} + \ln(\sin^2(x)) + \arctan(2x) + \tan(x)\csc(x)$$

#2. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(2a) (8 points) Let: $x + y^2 = \sin(xy)$. Use implicit differentiation to find $\frac{dy}{dx}$. Your answer may be an expression involving x and y.

(2b) (8 points) Let: $f(x) = x^{8\cos(x)}$. Use logarithmic differentiation to find f'(x). You don't have to simplify your final answer, but it should be a function of x only. #3. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know the answer to part (a), you should still attempt to find answer to part (b). Please be sure to justify your answers.

(3a) (8 points) A common inhabitant of human intestines is the bacterium Escherichia coli. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 51 cells. Find an expression for the number of cells after t hours. Your answer may involve logarithms and the number e.

(3b) (8 points) Element X is a radioactive element. A sample of element X decays at a rate proportional to the amount of mass in the sample. A sample of Element X has an initial mass of 10 grams. After exactly 20 days the sample has a mass of 5 grams. Find an expression for the mass m(t) of the sample after t days. Your answer may involve logarithms and the number e.

#4. (18 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(4a) (9 points) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.5 ft/s, how fast (in rad/s) is the angle (in radians) between the ladder and the ground changing when the bottom of the ladder is 6 ft from the wall? (That is, find the angle's rate of change when the bottom of the ladder is 6 ft from the wall.)

(4b) (9 points) A plane flying horizontally at an altitude of 3 miles and a speed of 400 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 miles away.

#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(5a) (8 points) Find the linear approximation of the function $f(x) = \sqrt{x-3}$ at x = 4. Use it to approximate the number $\sqrt{0.96}$

(5b) (8 points) Compute Δy and dy of function $y(x) = \frac{x}{1+x}$ when x = 1 and $dx = \Delta x = 0.1$.

#6. (18 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) A particle moves according to a law of motion given by the displacement function $s(t) = te^{-t}$, where $t \ge 0$ is measured in seconds and s is measured in feet. Find the total distance (in feet) traveled during the first 5 seconds.

(b) (8 points) If a ball is thrown vertically upward with an initial velocity of 10 ft/s, then its height (in feet) after t seconds is $s = 10t - t^2$. (Consider up to be the positive direction.) What is the maximum height (in feet) reached by the ball?

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