November 9, 2022
7:00-9:00 p.m.

## Your Name (Last, First)

## Student ID Number

## Signature

## Section Number

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| Section | Instructor | Class Time | Section | Instructor | Class Time |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Manas Bhatnagar | MWF 12:20-1:10pm |  | 11 | Sean Hart |
| 3 | Vefa Goksel | MWF 11:15-12:05pm | 12 | Garyfallia Katsimiga | TuTh 10:00-11:15am |
| 5 | Manas Bhatnagar | MWF 1:25-2:15pm |  | 13 | Garyfallia Katsimiga |
| TuTh 8:30-9:45am |  |  |  |  |  |
| 6 | Catherine Benincasa | MW 2:30-3:45pm | 14 | Carolyn Broz | TuTh 2:30-3:45pm |
| 7 | Jinguo Lian | MWF 9:05-9:55am | 16 | Sean Hart | MW 2:30-3:45pm |
| 8 | Jinguo Lian | MWF 10:10-11:00am | 17 | Richard Buckman | MWF 9:05-9:55am |
| 9 | Richard Buckman | MWF 10:10-11:00am | 18 | Aubain Nzokem | TuTh 2:30-3:45pm |
| 10 | Kevin Sackel | TuTh 1:00-2:15pm |  | 19 | Kevin Sackel |
|  |  | 20 | TuThu 8:30-9:45am |  |  |
|  |  |  | Carolyn Broz | TuTh 4:00-5:15pm |  |

- Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions and 14 pages. Please check that you have all pages, that they are consecutive, and that there are no duplicates. Otherwise, please raise your hand and let proctors know. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (Answers given without proper justification may receive no credit.)
- Be ready to show your UMass ID card when you hand in your exam booklet.

| QUESTION | PER CENT | SCORE |
| :---: | :---: | :---: |
| 1 | 16 |  |
| 2 | 16 |  |
| 3 | 16 |  |
| 4 | 18 |  |
| 5 | 16 |  |
| 6 | 16 |  |
| Free | 2 |  |
| TOTAL | 100 |  |

\#1. (16 points) Find the derivatives of the following functions. You do NOT need to simplify your answer. BTW, these two problems are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to solve part (b).
(1a) (8 points) $f(x)=\pi^{x} x^{100}+\frac{1-x^{2}}{1+x^{3}}+\log _{3}\left(\cot ^{2}(x)\right)+\arcsin (2 x)$.
(1b) (8 points) Find the derivative of $g(x)$. You do NOT need to simplify.

$$
g(x)=x^{3} \sqrt[3]{x}+\ln \left(\sin ^{2}(x)\right)+\arctan (2 x)+\tan (x) \csc (x)
$$

\#2. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(2a) (8 points) Let: $x+y^{2}=\sin (x y)$. Use implicit differentiation to find $\frac{d y}{d x}$. Your answer may be an expression involving $x$ and $y$.
(2b) (8 points) Let: $f(x)=x^{8 \cos (x)}$. Use logarithmic differentiation to find $f^{\prime}(x)$. You don't have to simplify your final answer, but it should be a function of $x$ only.
\#3. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know the answer to part (a), you should still attempt to find answer to part (b). Please be sure to justify your answers.
(3a) (8 points) A common inhabitant of human intestines is the bacterium Escherichia coli. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 51 cells. Find an expression for the number of cells after $t$ hours. Your answer may involve logarithms and the number e.
(3b) (8 points) Element X is a radioactive element. A sample of element X decays at a rate proportional to the amount of mass in the sample. A sample of Element X has an initial mass of 10 grams. After exactly 20 days the sample has a mass of 5 grams. Find an expression for the mass $m(t)$ of the sample after $t$ days. Your answer may involve logarithms and the number e.
\#4. (18 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(4a) (9 points) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $0.5 \mathrm{ft} / \mathrm{s}$, how fast (in rad/s) is the angle (in radians) between the ladder and the ground changing when the bottom of the ladder is 6 ft from the wall? (That is, find the angle's rate of change when the bottom of the ladder is 6 ft from the wall.)
(4b) (9 points) A plane flying horizontally at an altitude of 3 miles and a speed of $400 \mathrm{mi} / \mathrm{h}$ passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 miles away.
\#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(5a) (8 points) Find the linear approximation of the function $f(x)=\sqrt{x-3}$ at $x=4$. Use it to approximate the number $\sqrt{0.96}$
(5b) (8 points) Compute $\Delta y$ and $d y$ of function $y(x)=\frac{x}{1+x}$ when $x=1$ and $d x=\Delta x=0.1$.
\#6. (18 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(a) (8 points) A particle moves according to a law of motion given by the displacement function $s(t)=t e^{-t}$, where $t \geq 0$ is measured in seconds and $s$ is measured in feet. Find the total distance (in feet) traveled during the first 5 seconds.
(b) (8 points) If a ball is thrown vertically upward with an initial velocity of 10 $\mathrm{ft} / \mathrm{s}$, then its height (in feet) after t seconds is $s=10 t-t^{2}$. (Consider up to be the positive direction.) What is the maximum height (in feet) reached by the ball?

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