UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 1** **Oct. 6th, 2022**
7:00-9:00 p.m.

Your Name (Last, First)_____

Student ID Number_____

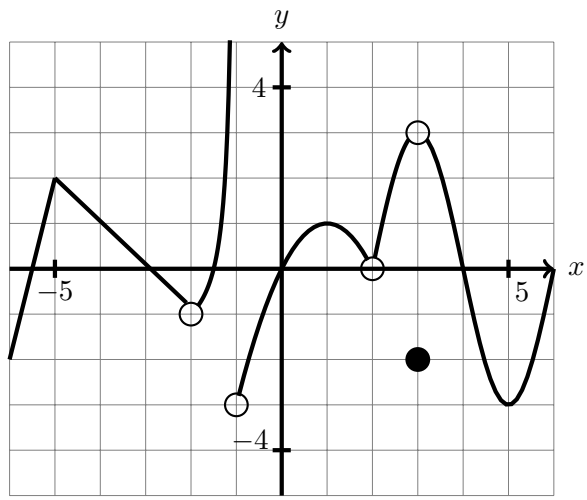
Signature_____ **Section Number** _____

Section	Instructor	Class Time		Section	Instructor	Class Time
1	Manas Bhatnagar	MWF 12:20-1:10pm		11	Sean Hart	MW 4:00-5:15pm
3	Vefa Goksel	MWF 11:15-12:05pm		12	Garyfallia Katsimiga	TuTh 10:00-11:15am
5	Manas Bhatnagar	MWF 1:25-2:15pm		13	Garyfallia Katsimiga	TuTh 8:30-9:45am
6	Catherine Benincasa	MW 2:30-3:45pm		14	Carolyn Broz	TuTh 2:30-3:45pm
7	Jinguo Lian	MWF 9:05-9:55am		16	Sean Hart	MW 2:30-3:45pm
8	Jinguo Lian	MWF 10:10-11:00am		17	Richard Buckman	MWF 9:05-9:55pm
9	Richard Buckman	MWF 10:10-11:00am		18	Aubain Nzokem	TuTh 2:30-3:45pm
10	Kevin Sackel	TuTh 1:00-2:15pm		19	Kevin Sackel	TuThu 8:30-9:45am
				20	Carolyn Broz	TuTh 4:00-5:15pm

- Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (**Answers given without proper justification may receive no credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	16	
4	16	
5	16	
6	18	
Free	2	
TOTAL	100	

#1. (16 points) Below is the graph of $f(x)$. Find the following function values or limits. If any of the quantities do not exist, clearly explain why.



(a) $f(3) =$

(b) $\lim_{x \to 3^-} f(x) =$

(c) $\lim_{x \to 3^+} f(x) =$

(d) $\lim_{x \to 3} f(x) =$

(e) $\lim_{x \to -1^+} f(x) =$

(f) $\lim_{x \to -1^-} f(x) =$

(g) $\lim_{x \to -1} f(x) =$

(h) $\lim_{x \to 2} f(f(x)) =$

#2. (16 points) Find the following limits. Please remember to justify all your answers, but do NOT use a graph or a table of values.

(2a) (5 points) $\lim_{x \to 5} \left(\frac{x^3 - 125}{x - 5} \right)$.

(2b) (6 points) $\lim_{x \to 3} \left(\frac{\sqrt{x^2+2} - \sqrt{11}}{x-3} \right)$.

(2c) (5 points) $\lim_{x\to\infty}\left(\frac{\sqrt{9x^2+100}}{1-3x}\right)$

#3. (16 points)

(3a) (8 points) Let

$$f(x) = \begin{cases} cx^2 + 9x, & \text{if } x < 1; \\ x^3 - cx, & \text{if } x \geq 1. \end{cases}$$

For what value of the constant c is the function $f(x)$ continuous on $(-\infty, \infty)$?

(3b) (8 points) Determine whether the function $f(x)$ defined below is discontinuous. If so, then where? Is it continuous from the right or continuous from the left there? Either way, justify all your answers.

$$f(x) = \begin{cases} 3^x, & x \leq 1; \\ x^3, & x > 1. \end{cases}$$

#4. (16 points)

(4a) (8 points) Let $f(x) = 2x + 5$. For any $\epsilon > 0$, find the largest value of δ such that if $|x + 1| < \delta$, then $|f(x) - 3| < \epsilon$. Express your answer in terms of ϵ.

(4b) (8 points) Given that $-e^{-x} \leq e^{-x}\sin(x) \leq e^{-x}$ for all x, determine $\lim_{x\to\infty} e^{-x}\sin(x)$. Name any theorem(s) that you are using, explain any preconditions and why they are satisfied. Make sure the logical flow is clear.

#5. (16 points) Please justify all your work. Name any theorems that you might be using to solve the following two problems, explain any preconditions and why they are satisfied.

(5a) (8 points) Let: $f(x) = 2^x - x^4$. Show that equation $f(x) = 0$ has a solution on $(0, 2)$.

(5b) (8 points) Find all asymptotes (whether vertical or horizontal) of the following function: $f(x) = \dfrac{x^2+1}{x^2-1}$.

#6. (18 points)

(6a) (12 points) Use the limit definition of the derivative to find $f'(x)$ for:

$f(x) = x^3 - x + 1.$

(6b) (6 points) Use the information from (a) to determine the slope of the line tangent to $f(x)$ at $x = 1$ and write the equation of the tangent line.

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