Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Instructor's Name _______________ **Section (01, 02, 03, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 13th, 2021**
1:00-3:00 pm

Instructions

- Please turn off and put away all electronic devices. This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions. Please do all your work in this exam booklet. You may continue to work on the back of the pages containing the problems (as well as the blank page at the end of this exam booklet), but – if you do so – please clearly state that you have done so on the page containing the given problem.
- Show all of your work, and be sure to organize it well. (**Answers given without proper justification may receive 0 credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	18	
3	16	
4	16	
5	16	
6	16	
Free	2	2
TOTAL	100	

#1. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) Let $f(x) = \sqrt{x}$. Use the definition of a derivative to find $f'(x)$.

(b) (8 points) A bacterial culture initially contains 200 cells and grows at a rate proportional to its size. After an hour the population has increased to 1000. Find an expression for the number of bacteria after t hours. Your answer may involve exponentials and/or logarithms.

#2. (18 points) Evaluate the following limits:

(a) (6 points) $\lim_{x\to 0} \frac{e^{2x} - 1 - 2x}{x^2}$.

(b) (6 points) $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^x$.

(c) (6 points) $\lim_{x\to\infty} x \sin \frac{4\pi}{x}$.

#3. (16 points) Let $f(x) = 2x^3 + 3x^2 - 120x$. We know that f is defined for all real numbers and that: $f'(x) = 6x^2 + 6x - 120$, $f''(x) = 12x + 6$.

(a) (8 points) Find the interval(s) on which f is increasing. Find the interval(s) on which f is decreasing. Determine the x-coordinates of all local maxima and local minima.

(b) (8 points) Find the interval(s) where the function is concave up. Find the interval(s) where the function is concave down. Determine the x-coordinates of all points of inflection of the graph of f(x).

#4. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) A rectangle has an area of 100 square meters. Determine the dimensions that minimize the perimeter, and find the minimum possible perimeter.

(b) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = x^2 - 2x + 7$ on the interval $[0, 2]$.

#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) A particle is moving with the velocity given by: $v(t) = t^2 - 6\sqrt{t}$. Given that $s(4) = 9$, find the position of the particle, $s(t)$.

(b) (8 points) Suppose that $1 \leq f'(x) \leq 2$ for all values of x. What are the minimum and maximum possible values of $f(10) - f(9)$?

#6. (16 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (8 points) Suppose that $\int_0^6 f(x)dx = 6$, $\int_0^8 g(x)dx = 4$ and $\int_0^8 [4f(x) + 5g(x)]dx = 56$. Find $\int_6^8 f(x)dx$.

(b) (8 points) Evaluate the Riemann sum for $f(x) = 1 - x$, $-1 \leq x \leq 2$ with three subintervals, taking the sample points to be the midpoints.

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