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Signature $\qquad$

Instructor's Name $\qquad$ Section (01, 02, 03, etc.) $\qquad$

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131
Exam 2
November 3th, 2021
7:00-9:00 p.m.

## Instructions

- Please turn off and put away all electronic devices. This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions. Please do all your work in this exam booklet. You may continue to work on the back of the pages containing the problems (as well as the blank page at the end of this exam booklet), but - if you do so - please clearly state that you have done so on the page containing the given problem.
- Show all of your work, and be sure to organize it well. (Answers given without proper justification may receive 0 credit.)
- Be ready to show your UMass ID card when you hand in your exam booklet.

| QUESTION | PER CENT | SCORE |
| :---: | :---: | :---: |
| 1 | 8 |  |
| 2 | 18 |  |
| 3 | 18 |  |
| 4 | 18 |  |
| 5 | 18 |  |
| 6 | 18 |  |
| Free | 2 |  |
| TOTAL | 100 |  |

\#1. (8 points) For each of the two problems below, please circle the correct answer. BTW, these two problems are NOT related and can be solved independently from each other. If you don't know the answer to part (a), you should still attempt to find answer to part (b). Please be sure to justify your selection.
(a) (4 points) A bacterial culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 500 . Find an expression for the number of bacteria after $t$ hours.
[A] $P(t)=100(2)^{t}$
[B] $P(t)=100(3)^{t}$
[C] $P(t)=100(4)^{t}$
[D] $P(t)=100(5)^{t}$
[E] $P(t)=100(6)^{t}$
(b) (4 points) A common inhabitant of human intestines is the bacterium Escherichia Coli. A cell of this bacterium in a nutrient-broth medium divides into two cells every 30 minutes. The initial population of a culture is 100 cells. When (in hours) will the population reach 10,000 cells?
[A] $t=\frac{\ln 100}{\ln 2}$
[B] $t=\frac{\ln 100}{\ln 4}$
[C] $t=\frac{\ln 100}{\ln 8}$
[D] $t=\frac{\ln 100}{\ln 16}$
[E] $t=\frac{\ln 100}{\ln 32}$
\#2. (18 points) Find the derivatives of the following functions. You do NOT need to simplify your answer. BTW, these two problems are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to solve part (b).
(a) (9 points) $f(x)=2^{x} x^{\pi}+\frac{1-e^{-x}}{1+e^{-x}}+\log _{2}(\sin (x))+\arcsin (x)+e^{5}$.
(b) (9 points) $g(x)=e^{2 x} \sqrt{x}+\ln (2 \cos (x))+\arctan (x)+\tan (x) \sec (x)+\pi^{4}$
\#3. (18 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(a) (9 points) Let: $5 x+y=1+x^{2} y^{2}$. Use implicit differentiation to find $\frac{d y}{d x}$. Your answer may be an expression involving $x$ and $y$.
(b) (9 points) Let: $f(x)=(x+1)^{x}$. Use logarithmic differentiation to find $f^{\prime}(x)$. You don't have to simplify your final answer, but it should be a function of $x$ only.
\#4. (18 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(a) (9 points) Find the average rate of change of the area of a circle with respect to its radius $r$ as r changes from 2 to 3 . Find the instantaneous rate of change when $r=3$. (Your answers may involve the number $\pi$.)
(b) ( 9 points) If a snowball melts so that its surface area ( $S=4 \pi r^{2}$ ) decreases at a rate of $2 \mathrm{~cm}^{2} / \mathrm{min}$, find the rate (in $\mathrm{cm} / \mathrm{min}$ ) at which the radius decreases when the radius is 5 cm .
\#5. (18 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(a) (9 points) Find the linear approximation of the function $f(x)=\sqrt{1-x}$ at $a=0$. Use it to approximate the number $\sqrt{0.95}$
(b) (9 points) Compute $\Delta y$ and $d y$ of function $y(x)=x^{2} 2^{x}$ when $x=1$ and $d x=\Delta x=0.1$.
\#6. (18 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).
(a) (9 points) A particle moves according to a law of motion given by the displacement function $s(t)=t^{2}-3 t$, where $t \geq 0$ is measured in seconds and $s$ is measured in feet. Find the total distance (in feet) traveled during the first 5 seconds.
(b) (9 points) A cylindrical tank with radius $r=5 m$ is being filled with water at a rate of $3 \mathrm{~m}^{3} / \mathrm{min}$. If volume of water in that tank is given by $V=\pi r^{2} h$ (where $h$ is the level of the water measured in meters), how fast is the level of the water in the tank increasing?

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