

Your Name (Last, First) _____ ID # _____

Signature _____

Your Instructor's Name _____ Section (01, 02, 03, etc.) _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131

Exam 1

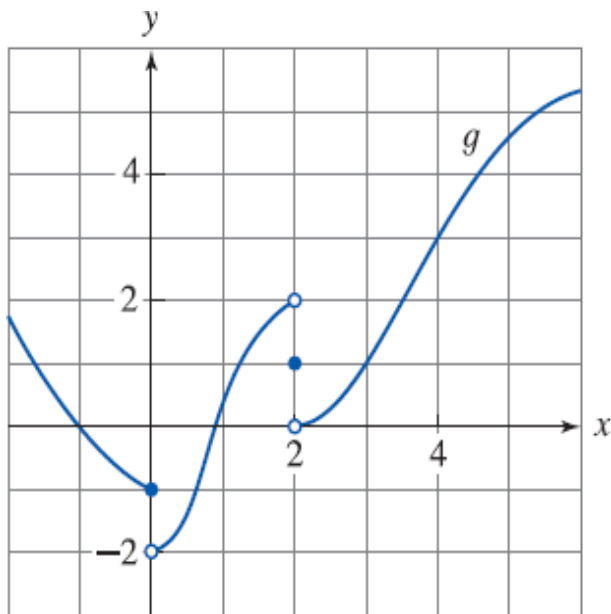
Sept. 29th, 2021
7:00-9:00 p.m.

Instructions

- Please turn off and put away all electronic devices. This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions. Please do all your work in this exam booklet. You may continue to work on the back of the pages containing the problems (as well as the blank page at the end of this exam booklet), but – if you do so – please clearly state that you have done so on the page containing the given problem.
- Show all of your work, and be sure to organize it well. (**Answers given without proper justification may receive 0 credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	16	
4	16	
5	16	
6	18	
Free	2	
TOTAL	100	

#1. (16 points) Below is the graph of $f(x)$. Find the following function values or limits. If any of the quantities do not exist, clearly explain why.



(a) $f(2) =$

(b) $\lim_{x \rightarrow 2^-} f(x) =$

(c) $\lim_{x \rightarrow 2^+} f(x) =$

(d) $\lim_{x \rightarrow -2^-} f(x) =$

(e) $\lim_{x \rightarrow -2^+} f(x) =$

(f) $\lim_{x \rightarrow -2} f(x) =$

(g) $\lim_{x \rightarrow 2} f(x) =$

(h) $\lim_{x \rightarrow 4} f(f(x)) =$

#2. (16 points) Find the following limits. Please remember to justify all your answers, but do NOT use a graph or a table of values.

(a) (5 points) $\lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x - 2} \right).$

(b) (6 points) $\lim_{x \rightarrow 1} \left(\frac{\sqrt{x^2 + 48} - 7}{x - 1} \right).$

(c) (5 points) $\lim_{x \rightarrow -2} \left(\frac{\frac{1}{2} + \frac{1}{x}}{2 + x} \right)$

#3. (16 points)

(a) (8 points) Find the value c such that the following limit exists and find out that limit.

$$\lim_{x \rightarrow -2} \frac{2x^2 + cx + c + 6}{x^2 + x - 2}$$

(b) (8 points) Find out if the following function $f(x)$ is discontinuous anywhere. If so, where? Is the function continuous from the right or continuous from the left there? As always, justify all your answers.

$$f(x) = \begin{cases} e^x, & x < 1; \\ x^2, & x \geq 1. \end{cases}$$

#4. (16 points)

(a) (8 points) Let $f(x) = 3x + \frac{3}{2}$. For any $\epsilon > 0$, find the largest value of δ such that if $|x + \frac{1}{2}| < \delta$, then $|f(x)| < \epsilon$. Express your answer in terms of ϵ .

(b) (8 points) Given that $\frac{10e^x-100}{2e^x} \leq f(x) \leq \frac{5\sqrt{x}}{\sqrt{x-1}}$ for all $x > 1$, determine $\lim_{x \rightarrow \infty} f(x)$. Name any theorem(s) that you are using.

#5. (16 points) Please justify all your work and precisely state any theorems that you might be using to solve the following two problems.

(a) (8 points) Let: $f(x) = -x^3 + 4x + 1$. Show that equation $f(x) = 0$ has a solution on $(-1, 0)$.

(b) (8 points) Find all the asymptotes (whether vertical or horizontal) of the following function: $f(x) = \frac{x^2 + 1}{x - x^2}$.

#6. (18 points)

(a) (12 points) Use the limit definition of the derivative to find $f'(x)$ for:

$$f(x) = \frac{1}{1 - \sqrt{x}}.$$

(b) (6 points) Use the information from (a) to determine the slope of the tangent line to $f(x)$ at $x = \frac{1}{4}$ and write down the equation of that tangent line.

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