

Your Name (Last, First) _____ ID # _____

Signature _____

Your Instructor's Name _____ Section (01, 02, 03, etc.) _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131

Exam 1

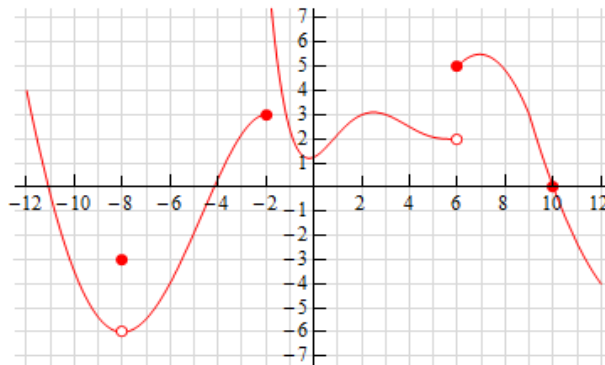
Sept. 23rd, 2020
7:00-9:00 p.m. EST

Instructions

- Please turn off and put away all electronic devices. This is a closed book exam. No calculators, notes, or books are allowed.
- There are six (6) questions. Please write each question on one page of paper, mark the question number and sub-question number clearly, and write your full name, student ID on top of each page of your solutions.
- After you have completed the exam, scan all pages of your solutions (one page per question) to a PDF file. You will have 30 minutes (9:00-9:30pm EST) after the exam is over to upload the PDF file to Gradescope. If, for any reason, you are unable to upload your solutions, please immediately email the PDF file containing your solutions to your instructor.
- Show all of your work, and be sure to organize it well. (**Answers given without proper justification may receive 0 credit.**)

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	16	
4	16	
5	16	
6	18	
Free	2	
TOTAL	100	

#1. (16 points) Below is the graph of $f(x)$. Find the following function values or limits. If a limit does not exist, please explain why.



(a) $f(-8) =$

(b) $\lim_{x \rightarrow -2^-} f(x) =$

(c) $\lim_{x \rightarrow -2^+} f(x) =$

(d) $\lim_{x \rightarrow 6^-} f(x) =$

(e) $\lim_{x \rightarrow 6^+} f(x) =$

(f) $\lim_{x \rightarrow -2} f(x) =$

(g) $\lim_{x \rightarrow 6} f(x) =$

(h) $\lim_{x \rightarrow -8} f(x) =$

#2. (16 points) Find the following limits. Please remember to justify all your answers.

(a) (5 points) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

(b) (6 points) $\lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x}$

(c) (5 points) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 3}}{1 - 2x}$

#3. (16 points) Find the value (or values) of x at which f is discontinuous and use limits to justify whether f is continuous from the right, or from the left or neither. Identify the discontinuity of the function as removable discontinuity, infinite discontinuity, or jump discontinuity.

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ (5 - x)^2 & \text{if } 0 < x < 2 \\ x^3 + 1 & \text{if } x \geq 2. \end{cases}$$

#4. (16 points) Please remember to justify all your answers.

(a) (8 points) Let $f(x) = 2x + 2$. For any $\epsilon > 0$, find the largest value of δ such that if $|x + 1| < \delta$, then $|f(x)| < \epsilon$. Express your answer in terms of ϵ .

(b) (8 points) Give that $7x \leq f(x) \leq 3x^2 + 4$ for all x in $(-\infty, 1)$, determine $\lim_{x \rightarrow 1^-} f(x)$.

#5. (16 points) Please justify all your work and precisely state any theorems that you use to solve the following problems:

(a) (8 points) Let $f(x) = e^x - x^2$. Show that the equation $f(x) = 0$ has a solution on $(-\infty, \infty)$.

(b) (8 points) Find all the asymptotes (whether vertical or horizontal) of the following function: $f(x) = \frac{x+1}{x-2}$.

#6. (18 points)

(a) (12 points) Use the limit definition of the derivative to find $f'(x)$ of the following function:

$$f(x) = 2x^2 + 6x + 1.$$

(b) (6 points) Use the information from (a) to find the slope of the tangent line to $f(x)$ at $x = 3$ and provide the equation for this tangent line.