Name (Last, First) ——————————————— ID # ———————————

Signature ———————————————————————————

Instructor ——————————————— Section (01, 02, 03, etc.) ———————

Math 131　　　　　　　Exam 2　　　　　　November 13, 2019
　　　　　　　　　　　　　　　　　　　　　　　　　　7:00-9:00 p.m.

Instructions

- Please turn off and put away all electronic devices.

- There are six (6) questions.

- Do all of your work in this exam booklet. You may continue work on the backs of pages and the blank page at the end, but if you do so, indicate where.

- No calculators, notes, or books are allowed.

- Show all of your work, and be sure to organize it well. (Answers given without supporting work may receive 0 credit.) Please **circle your final answers.**

- Be ready to show your UMass ID card when you hand in your exam booklet.

- **Good Luck!**

QUESTION	POINTS	SCORE
1	18	
2	16	
3	16	
4	16	
5	16	
6	16	
Free	2	
TOTAL	100	

#1. Find the derivatives of the following functions. You do **NOT** need to simplify your answers.

(a) (5 points)

$$f(x) = 3^x x^{100} + \cos(x) + \log_2(\sin(x))$$

(b) (5 points)

$$g(x) = \frac{e^x}{\ln(x)} + \tan(x) + \cot(x) + 3.14^\pi$$

(c) (8 points)

$$h(x) = \sec(x) + \csc(x) + \arctan(x) + \arcsin(x)$$

#2. A table of values for the functions $f(x), g(x)$, and $h(x)$, and for their first two derivatives is given below. You may assume that $f(x), g(x)$, and $h(x)$, and their first two derivatives exist at all real numbers.

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$	$h(x)$	$h'(x)$	$h''(x)$
1	0	1	2	-1	1	3	1	0	1
2	-1	2	3	1	-1	2	0	2	-1
3	2	3	-1	2	2	0	1	3	0

For each of the parts (a), (b), and (c), Justify your answers.

(a) (6 points) Let $P(x) = \dfrac{g(x)}{h(x)}$. Find $P'(1)$.

(b) (6 points) Let $Q(x) = f(x)g(x)$. Find $Q''(2)$

(c) (5 points) Let $R(x) = h(f(x))$. Find $R'(3)$.

#3.
(a) (8 points) Consider the curve $2x^2 + 3xy - y^2 = 5$. Use implicit differentiation to find $\dfrac{dy}{dx}$. Your answer may be an expression involving x and y.

(b) (8 points) Consider the curve $y = x^x$. Use logarithmic differentiation to find $\dfrac{dy}{dx}$. Express your answer in terms of x.

#4.

(a) (8 points) A bacterial culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 490. Find an expression for the number of bacteria after t hours. (Your answer may involve logarithms and the number e.)

(b) (8 points) Find the average rate of change of the area of a circle with respect to its radius r as r changes from 5 to 6. Find the instantaneous rate of change when $r = 5$. (Your answers may involve the number π.)

#5.

(a) (8 points) If a snowball melts so that its surface area $(S = 4\pi r^2)$ decreases at a rate of 5 cm^2/min, find the rate at which the diameter decreases when the diameter is 11 cm. (Your answer may involve the number π.)

(b) (8 points) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$. Use it to approximate the number $\sqrt{0.98}$.

#6.

    (a) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = x^2 - 2x + 3$ on the interval $[0, 2]$.

    (b) (8 points) Consider $f(x) = x^2$. Use an appropriate Theorem to show that there is a value of c in $(1, 2)$ such that $f'(c)$ is equal to the average rate of change of f on $[1, 2]$.

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