Name (Last, First) ______________________ **ID #** __________

Signature ______________________________________

Lecturer ____________________ **Section (01, 02, 03, etc.)** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 15, 2017**
1:00-3:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- You *may* use a calculator. If you do, be sure to show the set-up for what you are calculating and do *not* round intermediate results.
- Otherwise, this is a "closed-book" exam: do not use any books or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	15	
2	10	
3	15	
4	15	
5	15	
6	15	
7	15	
TOTAL	100	

#1. (15 points) The two parts of this question are unrelated. Make sure to show your work and circle your final answers. You cannot use a graph or a sequence of test points to justify your conclusions.

(a) (5 points) Evaluate the limit $\lim_{x\to\infty} \dfrac{x-9}{\sqrt{4x^2+3x+2}}$.

(b) Let $f(x)$ be the function $f(x) = \begin{cases} (2-x)^{\tan\left(\frac{\pi x}{2}\right)} & \text{if } x \neq 1 \\ e & \text{if } x = 1 \end{cases}$.

(i) (5 points) Evaluate the limit $\lim_{x\to 1} f(x)$.

(ii) (5 points) Is $f(x)$ continuous at $x = 1$? Use the definition of continuity to explain why or why not.

#2. (10 points) Both parts of this problem involve the function $f(x) = x^{2/5} + x^{4/5}$.

(a) (5 points) Using the Mean Value Theorem, show that there must be some $c > 0$ such that $f'(c) = \frac{20-0}{32-0}$. Make sure to justify that the theorem can be applied in this case.

(b) (5 points) Either use Rolle's theorem to show that there is some c such that $f'(c) = 0$, or else explain why this is not possible.

#3. (15 points) The two parts of this problem are unrelated. Make sure to circle your final answers. You do not need to simplify your answers at all.

(a) (8 points) Find the derivative of the function $f(x) = \tan\left(\sqrt{\frac{1-x}{1+x}}\right)$.

(b) (7 points) Use logarithmic differentiation to find the derivative of the function $g(x)$ defined below.

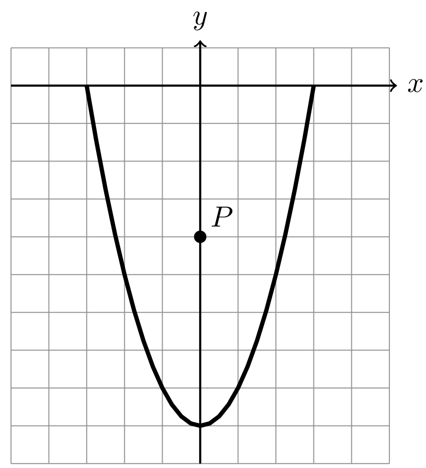
$$g(x) = \frac{\sqrt{x+10}}{(2x+3)^3(5x^3-1)^7}.$$

#4. (15 points) The two parts of this problem are unrelated. Make sure to circle your final answers.

(a) (7 points) Write a formula for the linearization of the function $f(x) = \arctan(x)$ at $a = 1$. Use this to estimate the value of $\arctan(1.06)$ rounded to three decimal places.

(b) (8 points) Two people ride their bicycles away from the same fixed point, leaving at the exact same time. One person rides due North at a speed of 5 m/s. The other rides due West at a speed of 3 m/s. At the moment exactly 10 minutes after they begin moving, what is the rate of change in the distance between the two people?

#5. (15 points) A small footpath is shaped like the parabola $y = x^2 - 9$ on the domain $[-3, 3]$. There is a statue located at the point $P = (0, -4)$.



Use calculus methods to find the coordinates of the points on the path that are closest to the statue and the coordinates of the points on the path that are farthest away from the statue. Make sure to carefully explain your reasoning.

#6. (15 points) All parts of this problem involve the function
$f(x) = (x^2 + 2x + 1)e^x$ on the domain $(-\infty, \infty)$.

(a) (3 points) Find the critical numbers of $f(x)$ on this domain.

(b) (3 points) Using calculus methods, list the intervals on which $f(x)$ is increasing and the intervals on which it is decreasing.

(c) (3 points) Using calculus methods, classify each critical number you found in part (a) as a local maximum, a local minimum, or neither.

This problem continues on the next page.

Continuation of problem #6 concerning $f(x) = (x^2 + 2x + 1)e^x$.

(d) (3 points) Using calculus methods, list the intervals on which $f(x)$ is concave up and list the intervals on which $f(x)$ is concave down.

(e) (3 points) Using calculus methods, find every x-value in this domain at which $f(x)$ has an inflection point.

#7. (15 points) All parts of this problem involve the function $g(x) = 3 + 2x - x^2$. Make sure to show all of your work and circle your final answers.

(a) (4 points) Write a formula for the most general antiderivative of $g(x)$.

(b) (4 points) Use a right-endpoint Riemann sum with $n = 3$ rectangles to estimate the area of the region under the curve $y = g(x)$ on the interval $[0, 3]$.

This problem continues on the next page.

Continuation of problem #7 concerning $g(x) = 3 + 2x - x^2$.

(c) (7 points) Write a definite integral representing the exact area of the region under the curve $y = g(x)$ on the interval $[0, 3]$. Then evaluate this integral using the definition of the definite integral as a limit of Riemann sums. You can use the following identities for sums of powers of consecutive natural numbers.

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

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