Name (Last, First) ______________________ **ID #** __________

Signature ______________________________________

Lecturer ____________________ **Section (01, 02, 03, etc.)** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 2** **November 13, 2017**
7:00-9:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are nine (9) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do **not** use a calculator, reference materials, or paper other than a booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	11	
2	11	
3	11	
4	11	
5	11	
6	11	
7	11	
8	11	
9	11	
FREE	1	1
TOTAL	100	

#1. Find the derivatives with respect to x of the following functions. Please **circle** your final answer. Show all work. You do **NOT** need to simplify your answers.

(a) (3 points) $f(x) = x^4 4^x$

(b) (4 points) $g(x) = \dfrac{e^x}{x^2 + e^2}$

(c) (4 points) $h(x) = \ln(\sin(e^{x^3}))$

#2. Let $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$. Please **circle** your final answers and show all work.

(a) (3 points) Find $f'(x)$.

(b) (4 points) Find all the critical numbers of the function $f(x)$ on the interval $(-\infty, \infty)$.

(c) (4 points) Find the absolute maximum and absolute minimum values of $f(x)$ on $[-1, 1]$.

#3.
(a) (5 points) Let $y = \sec(x)$. Using the derivative of $\cos(x)$, prove that $\dfrac{dy}{dx} = \sec(x)\tan(x)$.

(b) (6 points) Let $y = \ln(\sec(x))$ for x in $(0, \pi/2)$. Using the derivatives of $\ln(x)$, $\cos(x)$, or $\sec(x)$, prove that $\dfrac{dy}{dx} = \tan(x)$.

#4. Let $\arcsin(x)$ be the inverse function of $\sin(x)$, for $\sin(x)$ defined on $(-\pi/2, \pi/2)$. Please **circle** your final answers and show all work.

(a) (3 points) State the derivative of $\arcsin(x)$ with respect to x. You do not need to offer a proof.

(b) (8 points) $g(x) = \arcsin(\frac{\sin(x)}{2})$. Find $g'(x)$. You do **NOT** need to simplify your answer.

#5. A sample of a certain element has an initial mass of 8 mg. After 36 days, the sample has decayed to 1 mg. Please **circle** your final answers and show all work.

(a) (6 points) Find a formula for the remaining mass $m(t)$ (in mg) after t days. (*Hint*: The rate of change of the mass is proportional to the mass itself.) Your answer may involve exponentials and/or logarithms. You do **NOT** need to simplify your answers.

(b) (5 points) What is the half-life of this element? Your answer may involve exponentials and/or logarithms. You do **NOT** need to simplify your work.

#6. Consider the curve given by $y = e^{xy}$. Please **circle** your final answers and show all work.

(a) (7 points) Use implicit differentiation to find $\frac{dy}{dx}$. Your answer may be an expression involving x and y.

(b) (4 points) Find the equation of the tangent line to the curve at the point where $x = 0$.

#7. Let

$$f(x) = (1+x)^k$$

where k is a positive constant. Please **circle** your final answers and show all work.

(a) (4 points) Find the linearization $L(x)$ of $f(x)$ near $x = 0$.

(b) (4 points) Suppose you want to use $L(x)$ to find an approximation of the number $\sqrt{0.96}$. What number should k be, and what number should x be?

(c) (3 points) Approximate $\sqrt{0.96}$ using $L(x)$.

#8. (11 points) A certain gas obeys the equation $PV = 3T$, where P is the pressure in atmospheres (atm), V is the volume in liters (L), and T is the temperature in kelvins (K). Let time be measured in minutes (min). Suppose that, at a certain instant, $V = 1$ L and is decreasing at a rate of 2 L/min, and $T = 5$ K and is increasing at a rate of 4 K/min. What is rate of change of P with respect to time at this instant? Please **circle** your final answer and show all work.

#9. (11 points)

Let $f(x) = e^x$ defined for x in $[0, 1]$. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for $f(x)$ on [0,1], or explain why no such c exist.

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