Name (Last, First) ______________________ **ID #** ____________

Signature ____

Lecturer __________________ **Section (01, 02, 03, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 Exam 1 October 4, 2017 7:00-9:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- You *may* use a calculator. If you do, be sure to show the set-up for what you are calculating and do *not* round intermediate results.
- Otherwise, this is a "closed-book" exam: do not use any books or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	POINTS	SCORE
1	15	
2	15	
3	10	
4	20	
5	15	
6	10	
7	15	
TOTAL	100	

#1. A particle moves along the x-axis with equation of motion

$$s(t) = 3t - t^2 + 5$$

where s is measured in meters and t is measured in seconds.

(a) (5 points) Compute the average velocity of the particle on the interval $[1, 3]$.

(b) (5 points) Use a limit to compute the instantaneous velocity of the particle at time $t = 3$.

(c) (5 points) Does the particle ever come to a stop? (That is, is its instantaneous velocity ever exactly zero?) If so, at what time does this happen?

#2. Find the exact value of each limit without relying on a graph or table of values. If the value does not exist, indicate this by writing 'DNE'. Make sure to show all of your work.

(a) (5 points)

$$\lim_{t \to -1} \frac{\frac{3}{t} + 3}{t + 1}$$

(b) (5 points)

$$\lim_{x \to \infty} \frac{\sqrt{x^{10} - x}}{10 + 3x^5}$$

(c) (5 points)

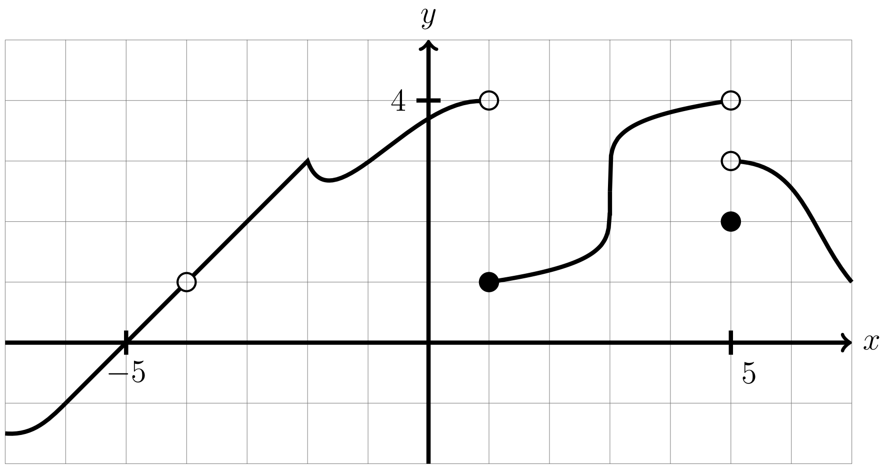
$$\lim_{x \to 0} \frac{x^2 + 2x}{x^2(x-1)(2x+4)}$$

#3. (10 points) The limit of $f(x) = \sqrt{x}$ as x approaches 9 is

$$\lim_{x \to 9} \sqrt{x} = 3.$$

Demonstrate this by finding the greatest possible value of δ corresponding to $\varepsilon = 0.01$ in the definition of the limit.
Note: a calculator is recommended for this problem, though not required. If you do use a calculator, you must carefully describe and explain your methods in order to receive full credit.

#4. Use the graph of a function $y = f(x)$ shown below to answer both parts of this question.



(a) (6 points) Find the value of each expression below, or write 'DNE' if it does not exist. You do not need to show your work.

$\lim_{x\to -4} f(x)$	$\lim_{x\to -2} f(x)$
$\lim_{x\to 1^-} f(x)$	$\lim_{x\to 1^+} f(x)$
$\lim_{x\to 5} f(x)$	$f(5)$

(b) (14 points) Use the graph to determine whether each statement is True or False, and circle the correct choice. You do not need to show your work.

At $x = -4$, $f(x)$ is continuous from the left.	True	False
At $x = -4$, $f(x)$ is continuous from the right.	True	False
At $x = -4$, $f(x)$ is continuous.	True	False
At $x = -4$, $f(x)$ is differentiable.	True	False
At $x = 1$, $f(x)$ is continuous from the left.	True	False
At $x = 1$, $f(x)$ is continuous from the right.	True	False
At $x = 5$, $f(x)$ is continuous from the left.	True	False
At $x = 5$, $f(x)$ is continuous from the right.	True	False
$f(x)$ is continuous on the interval $(-4, 1)$.	True	False
$f(x)$ is differentiable on the interval $(-4, 1)$.	True	False
$f(x)$ is differentiable on the interval $(1, 2)$.	True	False
$f(x)$ is differentiable on the interval $[1, 2)$.	True	False
$f(x)$ is differentiable on the interval $(1, 5)$.	True	False
$f(x)$ is differentiable on the interval $(1, 5]$.	True	False

#5. (15 points) Without relying on a graph, locate exactly all horizontal and vertical asymptotes of the function below. Explain (in terms of limits) why the asymptotes occur at these places.

$$y = \frac{x^3 + x^2}{(6x+6)(x^2-5)}.$$

Vertical asymptotes:

Horizontal asymptotes:

#6. (10 points) Use ideas and concepts from Calculus I to explain why we can be certain that the equation $x^4 - 4x - 1 = 0$ has at least two solutions.
Note: if you use any theorems, make sure to explain why any such theorems apply in this situation!

#7. Both parts of this problem concern the function

$$f(x) = \frac{1}{x^2}.$$

(a) (10 points) Use the **limit definition of the derivative** to find a formula for the derivative $f'(x)$.

(b) (5 points) Write an equation for the tangent line to $y = f(x)$ at $x = 2$.

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