Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer _________________________ **Section (01, 02, 03, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 19, 2016**
1:00-3:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- You *may* use a calculator. If you do, be sure to show the set-up for what you are calculating and do *not* round intermediate results.
- Otherwise, this is a "closed-book" exam: do not use any books or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	14	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
Free	2	2
TOTAL	100	

#1. A particle's position is given by

$$s(t) = 2t^3 + 3t^2 - 12t, \quad t \geq 0,$$

where t is measured in seconds and s in feet.

(a) (4 points) Find the velocity at time t.

(b) (3 points) When is the particle at rest?

(c) (3 points) When is the particle moving in the positive direction? Write your answer as a union of intervals.

(d) (4 points) What is the total distance travelled during the first 3 seconds? Simplify your answer.

#2. In this question, you may use your calculator to check your answer, but you should provide all details of your calculations. Answers without supporting work may receive no credit.

(a) (4 points) Evaluate $\lim_{x \to \infty} \dfrac{x^2 + 5x + 1}{3x^2 - 4x}$.

(b) (4 points) Evaluate $\lim_{x \to \infty} \sin\left(\dfrac{1}{x}\right)$.

(c) (6 points) Let

$$f(x) = \begin{cases} \dfrac{\sin(x)}{e^x - 1} & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$$

What value(s) of a, if any, will make f continuous at 0?

#3. Consider the function $f(x) = xe^{-x^2/2}$.

(a) (4 points) Find the derivative $f'(x)$.

(b) (4 points) Find all critical points of f. Show your work!

(c) (6 points) For each critical point, determine if it is a local minimum, local maximum, or neither. Justify your answer!

#4. For this question, let $f(x) = e^x$.

(a) (6 points) Find the linearization (i.e. linear approximation) $L(x)$ of $f(x)$ at 0. Use $L(x)$ to approximate $e^{0.1}$.

(b) (6 points) Suppose $Q(x) = a + bx + cx^2$ is a quadratic polynomial satisfying $Q(0) = f(0)$, $Q'(0) = f'(0)$ and $Q''(0) = f''(0)$, where $f(x) = e^x$, as in part (a). Solve for the constants a, b, c to find $Q(x)$.

(c) (2 points) Compute $Q(0.1)$. Write your answer in decimal notation. (This quadratic approximation is a better approximation to $e^{0.1}$ than the linear approximation in part (a).)

#5. The two parts of this problem are unrelated.

(a) (7 points) The height and base of a triangle are both changing over time. The height is increasing at a rate of 1 cm/min, while the area of the triangle is increasing at a rate of 3 cm^2/min. At what rate is the base of the triangle changing when the height is 10 cm and the base is 21 cm?

(b) (7 points) You are planning to make an open box from a square 6 by 6 inch piece of cardboard by cutting squares from the corners and folding up the sides. What is the largest volume possible for a box constructed in this way?

#6. Consider the function $f(x) = 4x^2 + \frac{1}{x}$ for **positive** values of x, i.e. for x in $(0, \infty)$. Suppose $G(x)$ is an antiderivative of f, i.e., $G'(x) = f(x)$.
(*Warning*: this question asks about both f and G; **do not mix them up**!)

(a) (4 points) Find the most general form of the antiderivative $G(x)$.

(b) (3 points) Find the x-values in $(0, \infty)$ for which $G(x)$ is increasing. Write your answer as a union of intervals.

(c) (4 points) Find the x-values in $(0, \infty)$ for which $f(x)$ is increasing. Write your answer as a union of intervals.

(d) (3 points) Find the x-values in $(0, \infty)$ for which $G(x)$ is concave-up. Write your answer as a union of intervals.

#7. Let $f(x) = \sqrt{1 - x^2}$.

(a) (6 points) Sketch the graph $y = \sqrt{1 - x^2}$. Sketch the rectangles corresponding to the Riemann sum for f over the interval $[-1, 1]$, with four approximating rectangles, using the *right* endpoints as sample points. Write a formula for this Riemann sum, and evaluate it.

(b) (3 points) Find the exact value of $\int_{-1}^{1} \sqrt{1 - x^2}\, dx$ by interpreting it in terms of area.

(c) (2 points) Is your answer to (a) an overestimate or an underestimate for $\int_{-1}^{1} \sqrt{1-x^2}\,dx$?

(d) (3 points) Suppose $\int_{-1}^{1} \left(\sqrt{1-x^2} + g(x)\right)\,dx = 5\pi$. Use part (b) to find $\int_{-1}^{1} g(x)\,dx$.

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