Name (Last, First) _____ **ID #** _____

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Lecturer _____ **Section (01, 02, 03, etc.)** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 2** **November 10, 2016**
7:00-9:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are ten (10) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do **not** use a calculator, reference materials, or paper other than a booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
TOTAL	100	

#1. Find the derivatives of the following functions. You do **NOT** need to simplify your answers.

(a) (3 points) $f(x) = 4x^3 - 5x^{-2}$

(b) (3 points) $g(x) = xe^x + \pi$

(c) (4 points) $h(x) = \dfrac{4x^3 - 5x^{-2}}{xe^x + \pi}$

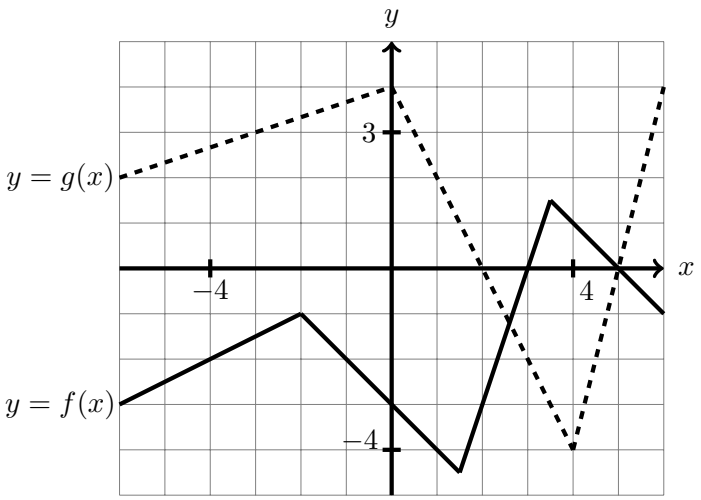
#2. For all parts of this question, let $f(x) = \dfrac{x+1}{x^2+3}$.

(a) Find the derivative $f'(x)$.

(b) Find all the critical numbers of the function $f(x)$.

(c) Find the absolute maximum and absolute minimum values of $f(x)$ on $[-1, 4]$.

#3. For all parts of this problem, let $f(x)$ and $g(x)$ be the functions whose graphs $y = f(x)$ and $y = g(x)$ are shown below, with solid and dashed lines, respectively.



(a) (4 points) Let $u(x) = f(x)g(x)$. Find $u'(1)$.

(b) (3 points) Let $v(x) = f(g(x))$. Find $v'(1)$.

(c) (3 points) Let $w(x) = f(f(x))$. Find $w'(1)$.

#4. Strontium-90 has a half-life of 28 days. A sample has an initial mass of 50 mg. (a) (6 points) Find a formula for the remaining mass $m(t)$ (in mg) after t days. (*Hints*: The rate of change of the mass is proportional to the mass itself. Your answer may involve exponentials and/or logarithms.)

(b) (4 points) How long will it take for this sample to decay to 2 mg?

#5. Consider the curve given by $y^2 - 4y = x^4 - 8x^2$.

(a) (5 points) Find the derivative $\dfrac{dy}{dx}$ (in terms of both x and y) at any point on this curve.

(b) (5 points) Find all points (x, y) where the tangent line to this curve is horizontal.

#6. (a) (5 points) Let $A(r)$ be the area of a disk (in cm^2) as a function of its radius r (in cm). Find the linearization (i.e. linear approximation) of $A(r)$ at radius 5 cm.

(b) (5 points) The radius of a circular disk is measured to be 5 cm, with a possible error in measurement up to 0.3 cm. Estimate the largest possible error in area.

#7. For all parts of this problem, let $f(x) = (x-3)^{-2}$.

(a) (3 points) Find the derivative of $f(x)$.

(b) (4 points) Show that there is no value of c in $(2, 4)$ such that $f'(c)$ is equal to the average rate of change of f on $[2, 4]$.

(c) (3 points) Explain why part (b) does not contradict the Mean Value Theorem.

#8. The position of a pendulum (in cm) after t seconds is given by $x(t) = 4\cos(\pi t)$.
(a) (4 points) Find the velocity $v(t)$, and find the acceleration $a(t)$.

(b) (2 points) Find the velocity and acceleration at the time $t = 0$.

(c) (4 points) Find the acceleration $a(t_0)$ at the time $t = t_0$ when the pendulum first swings through the midpoint of its motion (i.e., t_0 is the smallest positive t satisfying $x(t) = 0$).

#9. (10 points) A ladder of length 5 m leans against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 m/s, how fast (in radians per second) is the angle between the ladder and the ground changing when the ladder is 4 m from the wall?

#10. (a) (4 points) Let $f(x) = \ln\left(\dfrac{x^{6/5}(7+e^{4x})}{\sqrt{x^2+1}}\right)$. Find $f'(x)$.

(b) (3 points) Let $g(x) = \ln(\sin(x))$. Find $g'(x)$.

(c) (3 points) For the same function $g(x)$ as in part (b), find $\lim_{x\to 0^+}(xg'(x))$.

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