Name (Last, First) ______________________ **ID #** __________

Signature ______________________________________

Lecturer ___________________ **Section (01, 02, 03, etc.)** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 1** **October 5, 2016**
7:00-9:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- You *may* use a calculator. If you do, be sure to show the set-up for what you are calculating and do *not* round intermediate results.
- Otherwise, this is a "closed-book" exam: do not use any books or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	14	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
Free	2	2
TOTAL	100	

#1. In each part, find the limit. Justify your answers, but do **not** use a graph or table of values.

(a) (4 points) $\displaystyle\lim_{t\to 0}\left(\frac{3t^2+2}{t^2+1}\right)^4$

(b) (5 points) $\displaystyle\lim_{x\to -1}\frac{\sqrt{x^2+8}-3}{x+1}$

(c) (5 points) $\lim_{x \to 1^-} \frac{1-x}{\sqrt{1-x}}$

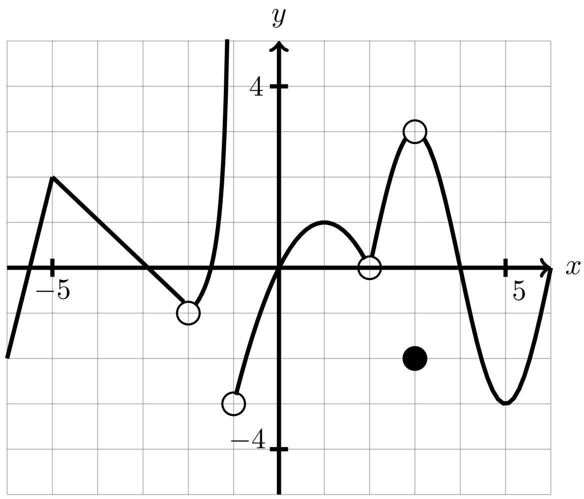
#2. Consider $f(x) = \dfrac{3x^2 - 6x}{x^2 - x - 2}$.

(a) (4 points) Find the domain of the function f.

(b) (5 points) By calculating relevant limits, determine the equations of all vertical asymptotes of the graph of $f(x)$. If there are none, explain why.

(c) (5 points) By calculating relevant limits, determine the equations of all horizontal asymptotes of the graph of $f(x)$. If there are none, explain why.

#3. Let $f(x)$ be the function whose graph $y = f(x)$ is shown below.



(a) (8 points) Find the following limits. Showing work is *not* required for part (a).

$\lim_{x\to 3} f(x) =$	$\lim_{x\to -2^+} f(x) =$
$\lim_{x\to -2^-} f(x) =$	$\lim_{x\to -2} f(f(x)) =$

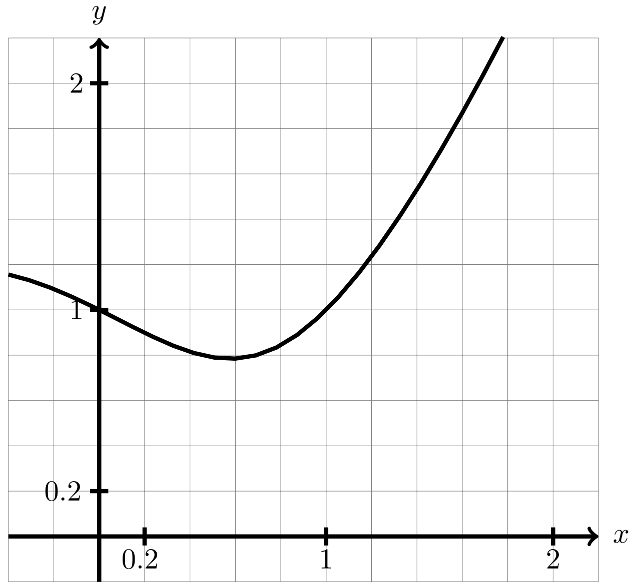
Note: The last limit involves the *composition* $f(f(x))$, and *not* simply $f(x)$.

(b) (3 points) Find $\lim_{x\to -5} \left(2f(x) + x^2 f(x)\right)$.

(Note that $f(x)$ is the function whose graph is shown on the previous page.)

(c) (3 points) Find $\lim_{x \to 2} f(x) \sin\left(\frac{1}{x-2}\right)$.

#4. Let $g(x)$ be the function whose graph $y = g(x)$ is shown below.



(a) (3 points) Find $\lim_{x \to 1} g(x)$.

(b) (3 points) Let L be your answer from part (a). Find a number $\delta > 0$ so that $|g(x) - L| < 0.4$ whenever $|x - 1| < \delta$.

(c) (8 points) Let $h(x) = 6x - 4$, and let $\varepsilon > 0$ be any positive number. Find a $\delta > 0$ in terms of ε so that

$$0 < |x - 5| < \delta \quad \text{implies} \quad |h(x) - 26| < \varepsilon.$$

(You will then have proven that $\lim_{x \to 5} h(x) = 26$.)

#5. Consider the function $f(x) = \dfrac{2 + 3x - x^3}{3 + e^{-x^2}}$.

(a) (2 points) Write down the domain of f.

(b) (2 points) At what x-values is f continuous?

(c) (4 points) Show that the equation $f(x) = 1$ has at least one solution in the interval $(0, 1)$. *Make clear which theorem(s) you use, if any.*

(d) (3 points) Find $\lim_{x \to \infty} f(x)$.

(Note that $f(x)$ is the function defined by the formula on the previous page.)

(e) (3 points) Use your answer in part (d) and proceed as in part (c) to show that $f(x) = 1$ has at least one solution in the interval $(1, \infty)$.

#6. Consider a function $f(x)$ with values given in the table below.

x	$f(x)$
4	2
3.5	1.125
3.25	1.015
3.1	1.001
3.01	1.000001
3	1

(a) (4 points) Find an equation for the line from (3,1) to (4,2).

(b) (4 points) Using the table of values of $f(x)$, find three successively better approximations for $f'(3)$.

(c) (3 points) From the approximations in part (c), guess this derivative $f'(3)$.

(d) (3 points) Find an equation for the tangent line to $y = f(x)$ at the point (3,1).

#7. Let $f(x) = x^2 - 3x$.

(a) (8 points) Use the *limit definition of the derivative* to find $f'(a)$ for any x-value a.

Show your work!

(b) (3 points) Find the positive x-value where the graph $y = f(x)$ crosses the x-axis.

(c) (6 points) Use part (a) to find the slope of the tangent line at the x-value you found in (b).

This page intentionally left blank