Homework Problems

1) Problems 2, 5 from section 2.2.
2) Problems 6, 7 from section 2.3

Practice Problems on 2.2-2.3

1. Find the solution to the diffusion equation which satisfies: \( u_t = 3u_{xx}, \)
\( u(x,0) = x^2 + 2x. \) Find where in the rectangle \( 0 \leq x \leq 2, \) \( 0 \leq t \leq T \) the maximum and minimum occur and identify that point in the \( x-t \) space-time plot.

2. Show that the wave equation has the following invariance properties, assuming that \( u(x,t) \) is a solution of
\[ u_{tt} = c^2 u_{xx} \]
- For \( y \) fixed, \( u(x-y,t) \) is also a solution of the wave equation.
- \( u_x(x,t) \) is also a solution of the wave equation.
- \( u(ax,at) \) for constant \( a \) is also a solution of the wave equation.

3. Consider the diffusion equation on \( (0,L) \) with Robin boundary conditions \( u_x(0,t) = a_0u(0,t) \) and \( u_x(L,t) = -a_1u(L,t) \) \( (a_0, a_1 > 0). \) Show that the boundary contributes to the decrease (in time) of \( \int_0^L u^2 dx. \)