Homework Problems

1) Problems 2, 5 from section 2.2.
2) Problems 6, 7 from section 2.3

Practice Problems on 2.2-2.3

1. Consider the wave equation (for $0 < x < L$)
   \[ u_{tt} = c^2 u_{xx} \]
   Using the energy, show that for $u(x,0) = 0$ and $u_t(x,0) = 0$, the only possible solution is $u(x,t) = 0$.

2. Show that the wave equation has the following invariance properties, assuming that $u(x,t)$ is a solution of
   \[ u_{tt} = c^2 u_{xx} \]
   - For $y$ fixed, $u(x - y, t)$ is also a solution of the wave equation.
   - $u_x(x,t)$ is also a solution of the wave equation.
   - $u(ax, at)$ for constant $a$ is also a solution of the wave equation.

3. Consider the diffusion equation on $(0, L)$ with Robin boundary conditions $u_x(0, t) = a_0 u(0, t)$ and $u_x(L, t) = -a_1 u(L, t)$ ($a_0, a_1 > 0$). Show that the boundary contributes to the decrease (in time) of $\int_0^L u^2 \, dx$. 