Homework Problems

1) Problems 7, 9 from Section 1.3; Problem 1 from 1.4.

2) Problems 1, 4 from Section 1.5; Problem 2 from 1.6.
   Notice that 9 in 1.3 and 4b in 1.5 are problems on the divergence theorem: use the appendix.

What Did We Learn in Ch.1 / Practice Problems

1. Order, Linearity, Homogeneity of a PDE.
   Problem: Classify (Order, and Linear Homogeneous, Linear Inhomogeneous or Nonlinear) the PDE:
   \[ u_t = (u^2)_x + u_{xxx} \]  \( \text{(1)} \)

2. How to solve linear transport equations / Meaning of Characteristics.
   Problem: Find the general solution of \( u_x - \sin(x)u_y = 0 \). Find the special solution that satisfies \( f(0,y) = e^y \).

3. The main second order equations (heat, wave, Laplace), where they come from and some of their properties.
   Problem: Show for the wave equation that there is an energy functional \( E = \frac{1}{2} \int_{-\infty}^{\infty} u_t^2 + c^2 u_x^2 \ dx \) such that \( dE/dt = 0 \) (provided that e.g., the derivative vanishes at \( x \to \pm \infty \)).

4. The significance/relevance of Initial/Boundary Conditions, what is well-posedness and how 2nd order PDEs are classified.
   Problem: Find the regions in 2d-space where \( x^2 u_{xx} + 4u_{xy} + y^2 u_{yy} = 0 \) is respectively elliptic, parabolic, hyperbolic. Plot these regions.