Homework Problems

• 1, 2, 9, 10, 11 from 5.3

Practice Problems on 5.2-5.3

1. Find the complex eigenvalues of the operator $d/dx$ subject to the boundary condition $X(0) = X(1)$. Are the eigenfunctions orthogonal on the interval $(0, 1)$?

2. Show that the operator $d^4/dx^4$ has no negative eigenvalues (in $(0, l)$) if supplemented with the boundary conditions $X(0) = X(l) = X''(0) = X''(l) = 0$.

3. Consider the rectangle with vertices $(0, 0), (0, 2), (1, 0)$ and $(1, 2)$. Solve the diffusion equation, with Dirichlet BC on the sides of the square and $u(x, y, 0) = 2$ for $0 < x < 1$ and $0 < y < 2$.

4. Show that the operator $\hat{H}u = -\frac{d^2 u}{dx^2} + V(x)u$ is Hermitian if supplemented with Neumann boundary conditions in $(a, b)$. Show it for real eigenfunctions and also for complex eigenfunctions.