Homework Problem Set 1

1. In a way similar to what we have shown in class, write a small matlab script that performs the algorithm of problem 11, Chapter 1 of Atkinson for converting positive decimals to hexadecimals. Apply the algorithm to find the hexadecimal representation of 490, 1270 and 12900.

2. Consider the integral (with $a > 2$)

$$I_n = \int_0^1 \frac{x^n}{a-x} dx.$$  

- Show that $0 < I_n < 1$ and $\lim_{n \to \infty} I_n = 0$.
- Find $I_0$ exactly.
- Derive a recursive formula for $I_n$ (Hint: it may help you to consider $I_n - aI_{n-1}$).
- Using the recursive relation, write a small Matlab script to obtain $I_{10}$ (using the exact value of $I_0$ you obtained) for $a = 10$.
- If the error in $I_0$ (the difference between the true value of $I_0$ i.e., $I_0^T$ and its approximate value $I_0^A$) is $\epsilon_0$, find the propagated error in computing $I_{10}$.
- Similarly (but backwards) if you know the error $\epsilon_{10}$, find the formula that gives the propagated error in computing $I_0$.
- If you were given the option of starting with an error $\epsilon_0$ and computing forward or that of starting with a (similar to $\epsilon_0$ in value) error $\epsilon_{10}$ and computing backward, which one would you pick and why?

3. Linear Systems

- Solve the system

$$\begin{align*}
x_1 + 2x_2 + 2x_3 &= 1 \\
4x_1 + 4x_2 + 12x_3 &= 12 \\
4x_1 + 8x_2 + 12x_3 &= 8,
\end{align*}$$

with Gaussian elimination using partial pivoting. As a byproduct, give the LU decomposition of the relevant matrix. Use the commands `lu(A)` and $x = A \backslash b$ to check your results in Matlab.
- Consider the Gauss-Jordan variant of Gaussian elimination (section 8.3 of Atkinson) and prove the estimate of (8.3.2) about the number of MD in it being $\approx n^3/2$.
- Find the Cholesky decomposition of $A = [4, 2, 1; 2, 3, 1; 1, 1, 4]$. Check the result using $[S,p] = chol(A)$. 