Homework Problems

1) 4.10, problem 9
\[ f = 10x^{-9} \Rightarrow F(x) = 10 \frac{1}{8} x^{-8} + C \text{ for any } x \neq 0 \]

2) 4.10, problem 23
\[ f''' = e^t \Rightarrow f''(t) = e^t + C \Rightarrow f'(t) = e^t + Ct + D \Rightarrow f(t) = e^t + Ct^2 + Dt + E. \]

3) 5.1, problem 14

For an increasing function, using left endpoints gives always an underestimate, while using right endpoints always results in an overestimate (convince yourselves of that!)

Let’s use \( n = 6 \) to get an estimate. \( \Delta t = (30-0)/6 = 5 \text{ sec} = 5/3600 = 1/720 \text{ hr}. \)

Then
\[ A \approx \frac{1}{720} [v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)] \]  \( \text{(1)} \)

Hence, substituting \( A \approx 0.725. \)

4) 5.3, problem 1

\( g(1) = 2 \times 1 = 2 \) (a rectangle); \( g(2) = 2 + 2 + 2 \times 1 \times 1/2 = 5 \) (a rectangle and a rectangle and a triangle) and similarly \( g(3) = 7 \) (add the area of another triangle) and \( g(6) = \ldots = 3 \) (add all the elementary areas – the negative ones with a (-) sign).

\( g \) is increasing between 0 and 3 because we keep adding area. It has a maximum value when we start subtracting area, hence at \( x = 3. \)

5) 5.3, problem 49
\[ y = \int_{\sqrt{2}}^{x} \sqrt{t} \sin(t)dt = \int_{1}^{\sqrt{x}} \sqrt{t} \sin(t)dt + \int_{\sqrt{x}}^{x} \sqrt{t} \sin(t)dt = -\int_{1}^{\sqrt{x}} \sqrt{\tau} \sin(t)dt + \int_{1}^{x} \sqrt{t} \sin(t)dt \]

Hence, from chain rule:
\[ y' = -x^{1/4} \sin(\sqrt{x}) \times \frac{d}{dx} \sqrt{x} + x^{3/2} \sin(x^3) \times \frac{d}{dx} x^3 = \ldots = 3x^{7/2} \sin(x^3) - \frac{1}{2} \sin(\sqrt{x})x^{-1/4} \]

6) 5.4, problem 10
\[ \int (x^2 + 1 + \frac{1}{1+x^2})dx = \frac{x^3}{3} + x + \tan^{-1} x + C \]
7) 5.4, problem 42

\[ f^2_0 (x^2 - |x - 1|)dx = f^1_0 (x^2 + x - 1)dx + f^2_1 (x^2 - x + 1)dx = \left[ \frac{x^3}{3} + \frac{x^2}{2} - x \right]_0^1 + \left[ \frac{x^2}{2} - \frac{x^2}{2} + x \right]^2 = \ldots = 5/3. \]

8) 5.4, problem 48

The integral, using the total change theorem, represents the change in charge from time \( t = a \) to time \( t = b \).