Stat705 PS 7. Due in class Monday, November 13th

1. Let $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}, \mathbf{X}$ is n by p, $p < n$, rank p.

   (a) Let $\mathbf{r} = (\mathbf{I} - \mathbf{P})\mathbf{y}$ where $\mathbf{P} = \mathbf{X}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}$. Let $\mathbf{U}\mathbf{D}\mathbf{U}^{\mathrm{T}}$ be the SVD of $(\mathbf{I} - \mathbf{P})$. What is $cov(\mathbf{U}^{\mathrm{T}}\mathbf{r})$? Show that p elements of $\mathbf{U}^{\mathrm{T}}\mathbf{r}$ are always zero.

   (b) Let p_{ii} be the ith diagonal element of $\mathbf{P}$. Use the Sherman-Morrison formula given in class to prove that the ith deleted residual is $r_i/(1 - p_{ii})$.

2. Assume the same model and notation as above. As given in class, Cook's distance is

$$D_i = \frac{\sum_{j=1}^{n}(\widehat{y}_j - \widehat{y}_{j(i)})^2}{pMSE}.$$

   (a) Describe in words what D_i means.

   (b) Show that

$$D_i = \frac{r_i}{pMSE}\left[\frac{p_{ii}}{(1 - p_{ii})^2}\right].$$