

Stat705 PS 2. Due in class September 22nd

NOTE 1: It is fine to work with other students on problem sets, and that is encouraged. Each person's write up must be done separately though, and identical answers (to relatively complicated problems) from different students will not be graded.

NOTE 2: If you use a source other than the textbook to do these problems, you must list the source. Additionally, it is never OK to copy from another source verbatim! If we notice that, the problem won't be graded.

Please read chapter 2 in Plane Answers...

1. Suppose $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \sim N(\mathbf{0}_2, \sigma^2 \mathbf{I})$. Are $y_1^2 + 2y_1y_2 + y_2^2$ and $y_1^2 - 2y_1y_2 + y_2^2$ independent? Why or why not?

2. $\mathbf{y} \sim N(\mathbf{0}, \Sigma)$. Suppose \mathbf{A} is symmetric and $\mathbf{B}\Sigma\mathbf{A} = \mathbf{0}$.

(a) Are $\mathbf{A}\mathbf{y}$ and $\mathbf{B}\mathbf{y}$ independent? Why or why not?

(b) Are $\mathbf{B}\mathbf{y}$ and $\mathbf{y}^T\mathbf{A}\mathbf{y}$ independent? Why or why not?

(c) Suppose \mathbf{B} is symmetric. Are $\mathbf{y}^T\mathbf{A}\mathbf{y}$ and $\mathbf{y}^T\mathbf{B}\mathbf{y}$ independent? Why or why not?

3. Consider the simple model $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \beta + \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$, with $E(e_i) = 0$, $\text{var}(e_i) = \sigma^2$, $i = 1, 2$. Note that β is a scalar.

(a) Draw the set $\left\{ \hat{\mathbf{y}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} c, \text{ for any } c \in R^1 \right\}$ and include the point $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ on the plot. (I'd make the x-axis y_1 and the y-axis y_2 .)

(b) Let $\mathbf{P} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} (1/10) \begin{pmatrix} 1 & 3 \end{pmatrix}$. Is \mathbf{P} idempotent? Why or why not? (Verify that \mathbf{P} is $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$.)

- (c) Consider the set $\{\mathbf{P}\mathbf{x}$ for any $\mathbf{x} \in R^2\}$. How does it compare to the set in (a)?
 - (d) Plot the point $\hat{\mathbf{y}} = \mathbf{P}\mathbf{y}$ on your graph. Verify by eye that it's the closest point to \mathbf{y} in the set from (a).
 - (e) Plot the set $\{(\mathbf{I}_2 - \mathbf{P})\mathbf{x}$ for any $\mathbf{x} \in R^2\}$ on your graph and include the point $\mathbf{r} = (\mathbf{I}_2 - \mathbf{P})\mathbf{y}$. Verify by eye that it's the closest point to \mathbf{y} in this set.
 - (f) What does $(\mathbf{I}_2 - \mathbf{P})\mathbf{P}$ equal and why does that make sense?
 - (g) Verify that $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{r}$.
4. The problem involving the integral I mentioned in class will come in a handout on Monday.