Estimability and Identifiability

Please read about estimability and identifiability Section 2.1 in Plane Answers. Please hand this in on Friday, October 6th. Also, please don't forget that (a) there is no class on Wed Oct 4th, (b) PS4 is not due until Tues Oct 10th.

Suppose $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}, \mathbf{X}$ is n by p, $n > p$.

Definition: $\lambda^{\mathrm{T}}\boldsymbol{\beta}$ is estimable if $\lambda^{\mathrm{T}}\boldsymbol{\beta} = \rho^{\mathrm{T}}\mathbf{X}\boldsymbol{\beta}$ for some ρ^{T}. In words, linear combinations of means of $\mathbf{y}$ are defined to be estimable. To match the book's notation, λ^{T} and ρ^{T} are individual rows of Λ^{T} and P^{T} respectively.

1. Suppose $\mathbf{X}$ has full rank and you are given a vector λ^{T}. Construct a ρ^{T} so that $\rho^{\mathrm{T}}\mathbf{X}\boldsymbol{\beta} = \lambda^{\mathrm{T}}\boldsymbol{\beta}$.

2. Now suppose that $\mathbf{X}$ doesn't have full rank. For instance, suppose $E(\mathbf{y}) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$. Find 4 (different) non-identifiable linear functions of $\boldsymbol{\beta}$ and 4 (different) identifiable functions. (Show that they are not identifiable and identifiable too.)