Stat705 Exam 1

1. Suppose $\mathbf{y} \sim MVN(\mathbf{0}, \mathbf{\Sigma}), \mathbf{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, -1 < \rho < 1$. Are $y_1^2 + 2y_1y_2 + y_2^2$ and $y_1^2 - 2y_1y_2 + y_2^2$ independent? Why or why not? You may cite a theorem without proof.

2. Let $\mathbf{y}$ be a random vector with length n. Let $\mathbf{X}$ be an n by p matrix with rank p. Suppose $\mathbf{y}$ is normal with mean $\mathbf{X}\boldsymbol{\beta}$ and covariance $\sigma^2\mathbf{I}_n$.

   (a) What is the least squares estimator of $\boldsymbol{\beta}$? (You do not need to derive this.)

   (b) What are matrix expressions for $\widehat{\mathbf{y}}$ and $\mathbf{e}$ (residuals)? (You do not need to derive this.)

   (c) What is the distribution of the vector $\mathbf{t} = (\widehat{\mathbf{y}}^{\mathrm{T}}, \mathbf{e}^{\mathrm{T}})^{\mathrm{T}}$? (Please justify your answer.)

   (d) Are SSE and $\widehat{\beta}$ independent? Why or why not?

3. Evaluate the integral $\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} (\mathbf{x}^{\mathrm{T}}\mathbf{B}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{c}) \exp\left\{-(\mathbf{x}-\mathbf{a})^{\mathrm{T}}\mathbf{\Sigma}^{-1}(\mathbf{x}-\mathbf{a})\right\} dx_1, \dots, dx_n$. For full credit, please justify each step.