Simulating the Single and Double Degenerate Channels of Type Ia Supernovae

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Outline

I. Properties of Type Ia Supernovae

II. Simulations of Single-Degenerate Channel

a. Full-Star Simulations

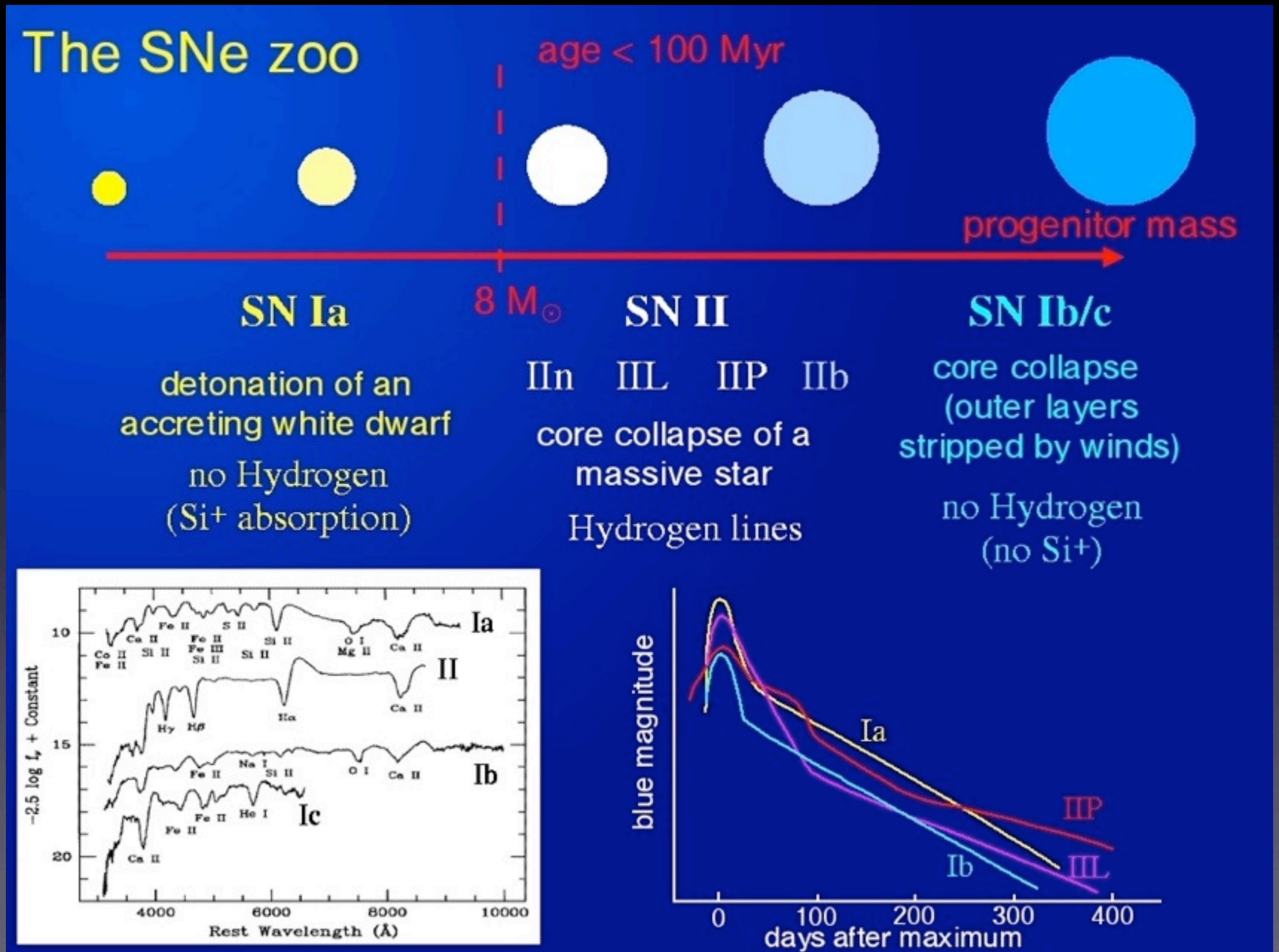
b. Verification Studies of Burning Flame Bubbles

III. Prospects for Simulating Double-Degenerate Channel

a. Post-Merger Phase over a Viscous Evolution Time

I. Observational Properties of Type Ia Supernovae

Supernovae Types



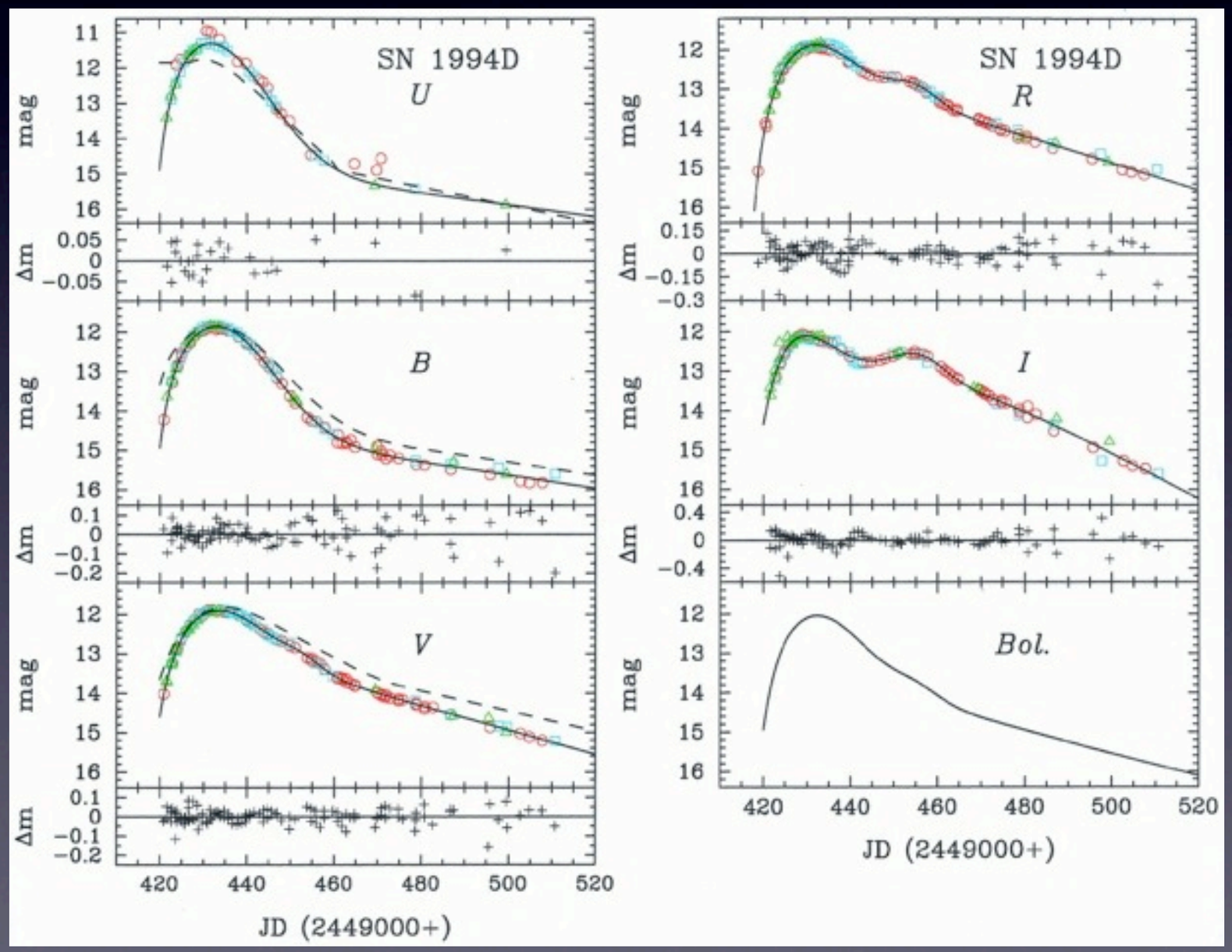
Cosmological Observations of Type Ia Supernovae

- The combination of the identification of the Branch normal population of Type Ia supernovae and advancements in observational technology in the 1980s and 90s made it feasible to employ Ia events as cosmological probes. (Zwicky, 1939; Colgate, 1979)

Hubble Key Project Team, High Z Supernovae Search Team

Type Ia Supernovae Light Curves

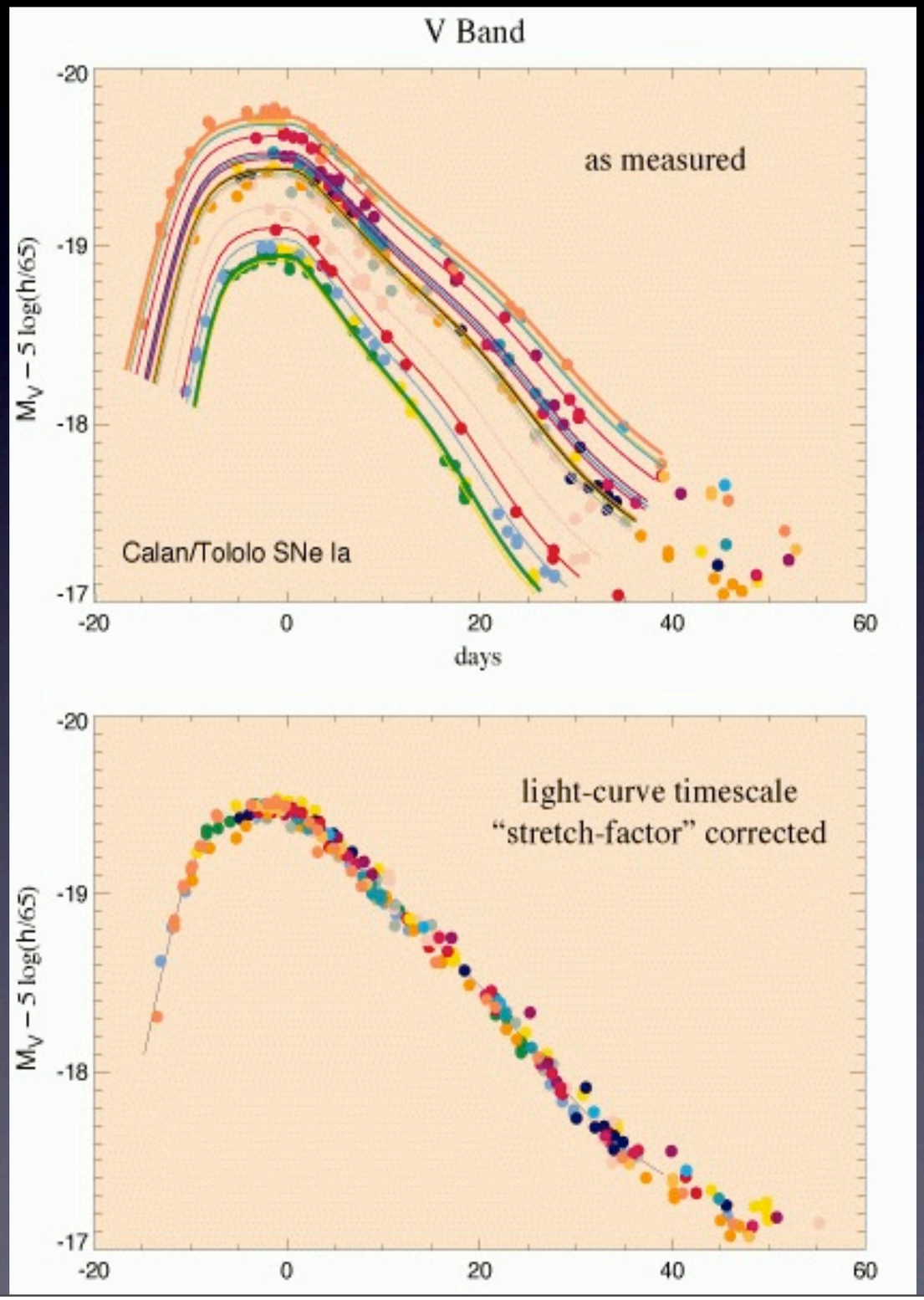
- Optical Type Ia light curves are powered by the decay of radioactive Ni-56 (half life of 6.077 days) and Co-56 (half life of 77.27 days) (Truran, 1969; Colgate & McKee, 1969)



Standardizing Light Curves - The Phillips Relation

- *Most* known Type Ia supernovae fit a universal light curve with a free parameter specified by the width of the curve. (Phillips, 1993)

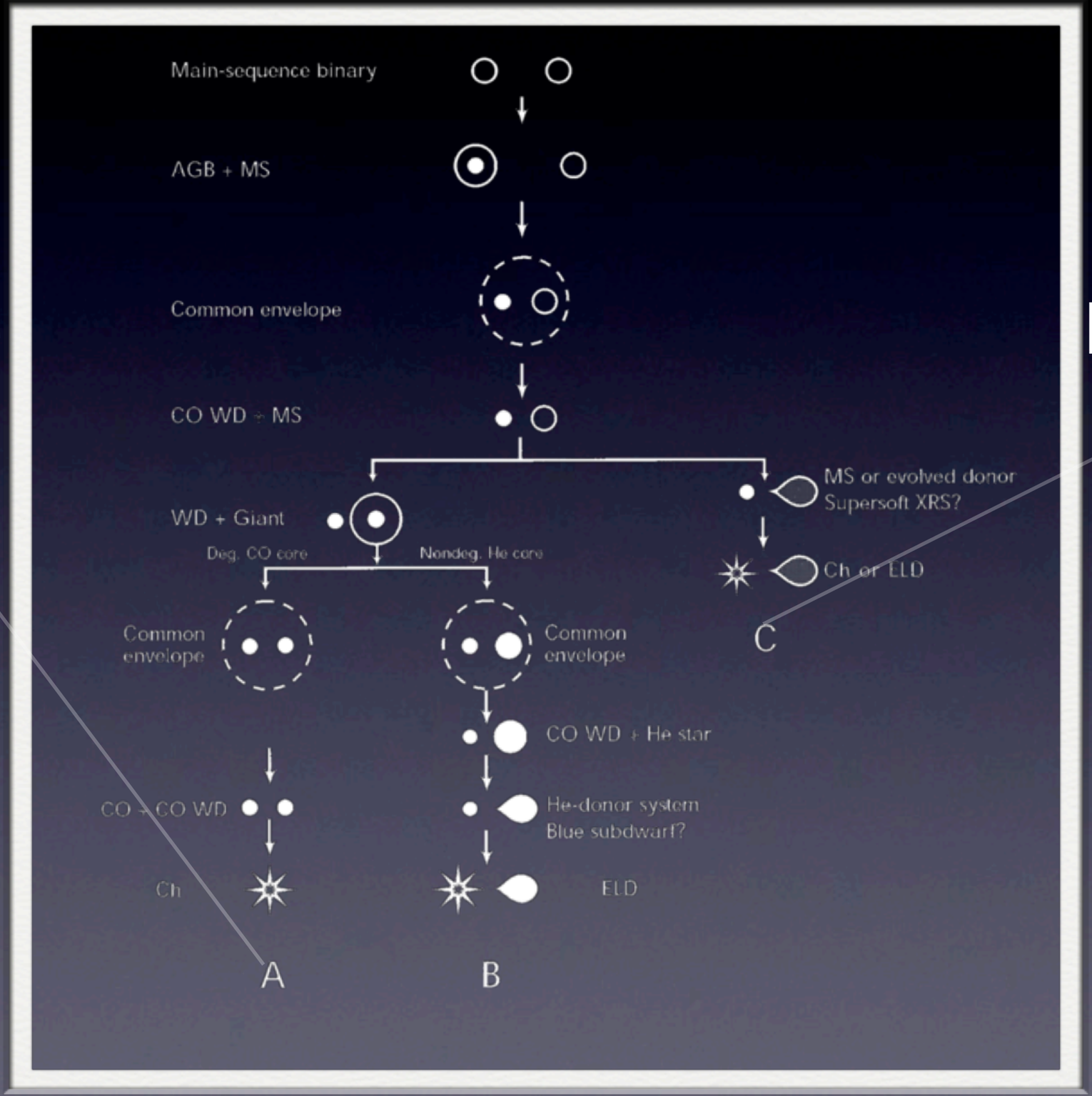
- The Phillips relation allows astronomers to use Type Ia events as *standardizable candles* of cosmological distances, and the characterization of *dark energy* (Riess *et al*, 1998; Perlmutter *et al*, 1999)



Single and Double Degenerate Channels of SNe Ia

Double Degenerate

Single Degenerate



Yungelson & Livio (2000)

White Dwarf Nuclear Energetics

- Fermi energy for *N* relativistic electrons in a fully degenerate white dwarf of radius *R*, number density *n*, simply estimated by Heisenberg and Pauli -

$$E_F \sim \hbar n^{1/3} c \sim \frac{\hbar N^{1/3} c}{R}$$

- Total energy is

$$E = N E_F + E_G \sim \frac{\hbar N^{4/3} c}{R} - \frac{G N^2 m_p^2}{R}$$

- Chandrasekhar mass is approximately

$$M_{\rm max} \sim \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{m_p^2} \sim \left(\frac{m_{\rm Planck}}{m_p}\right)^2 m_{\rm Planck} \sim 2\ M_\odot\ (1.4\ M_\odot)$$

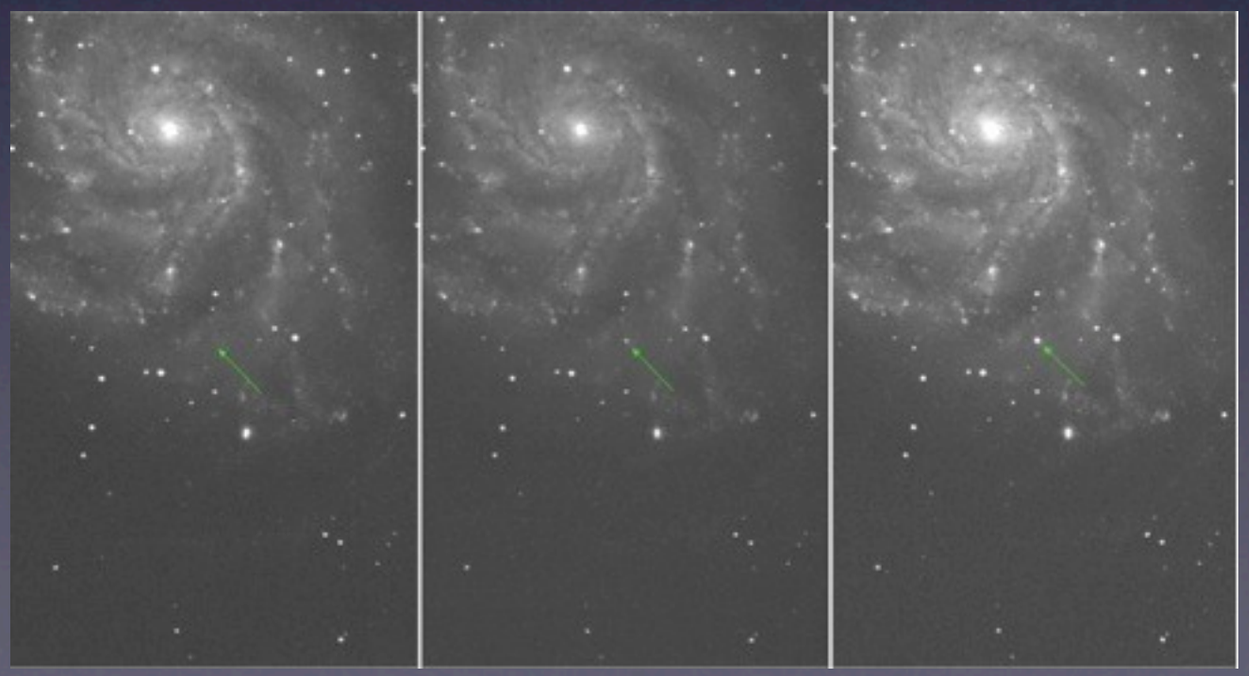
- Energy release $^{12}\mathrm{C} + ^{12}\mathrm{C}$ is $5 \cdot 10^{17}$ erg/g

$$1.4 \cdot 10^{51} \mathrm{erg} \sim 1.4 \text{ foe}$$

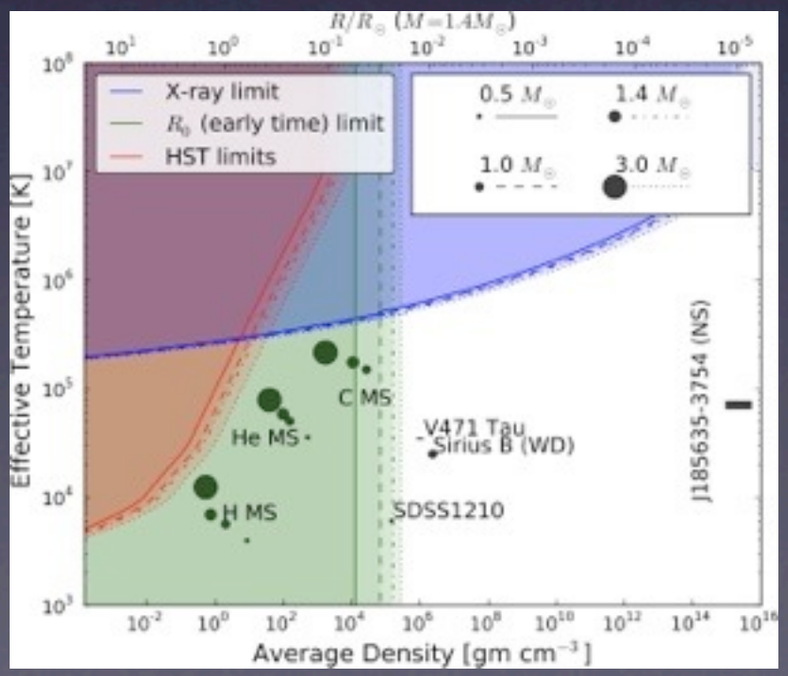
SN 2011fe

- On August 24, 2011 Palomar Transient Factory (PTF) discovered 2011fe in the pinwheel galaxy M101 at a distance of 21 million light years

- Combination of early-time light curves, X-ray and Hubble limits *directly* constrains the primary progenitor to a C/O white dwarf *for the first time* (Nugent et al, 2011; Bloom et al, 2012)



Nugent et al, 2011



Bloom et al, 2012

Challenges for SD and DD Channels

- For both channels - insufficient number of progenitor systems to account for observed rates (eg, Di Stefano 2010a,b)

- For SD - Absence of detectable H-alpha in normal SN 2005cf and subluminous SN 2005am. (Leonard, 2007; but also Justham, 2011, Ilkov & Soker, 2012)

- Also for SD - Steady-burning requires a narrow range of accretion rates (Townsley & Bildsten, 2003)

- For DD - why not an accretion-induced collapse to a neutron star? (Nomoto & Kondo, 1991, Saio & Nomoto, 1998, Saio & Nomoto, 2004)

Key Outstanding Theoretical Questions

- What mechanisms lead to Type Ia supernovae in both the single- and double-degenate channels?
- Under what conditions do we achieve a successful explosion? In double degenerate, why not an accretion-induced collapse?
- Are our models consistent with expected isotopic abundances, and nuclear energy yields?
- Can theoretical models yield useful predictions in X-ray and UV?

II. The Single Degenerate Channel

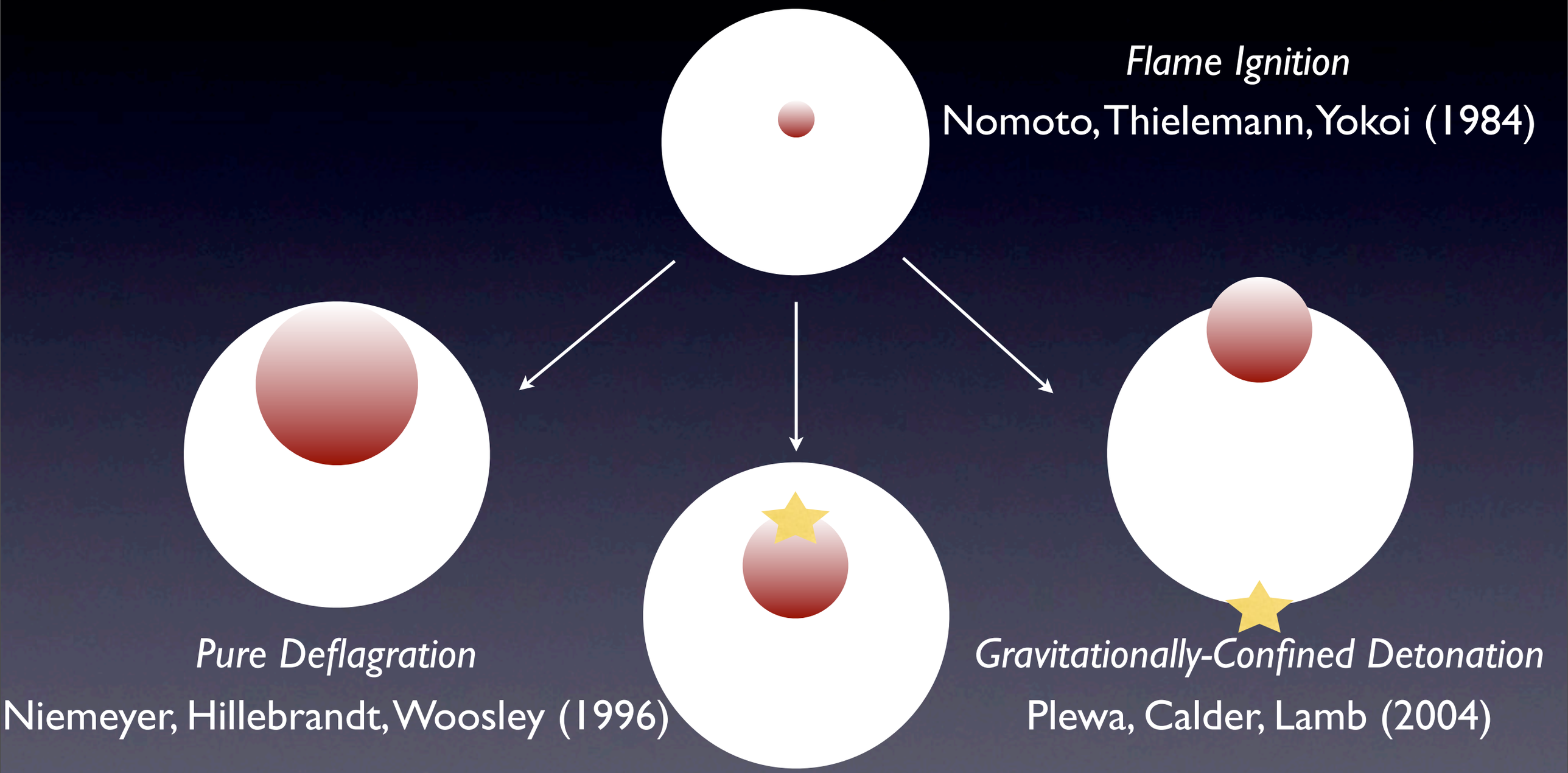
Single Degenerate Type Ia Progenitors : Chandrasekhar Mass, Relativistically-Degenerate C/O White Dwarfs

Central Burning

Radiative Envel

Convective Core

Single Degenerate Type Ia Supernovae Mechanisms



Rayleigh-Taylor Instability

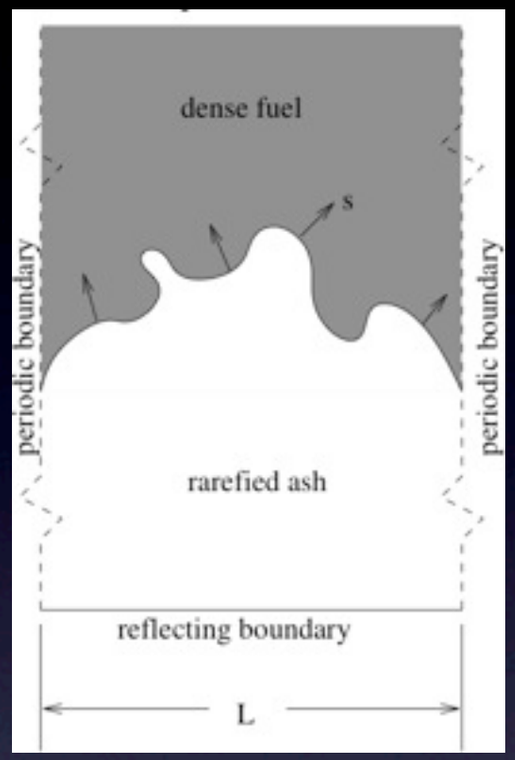
- Dense fuel overlaying rarefied ash is Rayleigh-Taylor unstable

- Classic Rayleigh-Taylor instability predicts linear

$$h(t) = h_0 \exp[(Agk)^{1/2} t]$$

$$A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

Smallest wavelengths grow fastest.



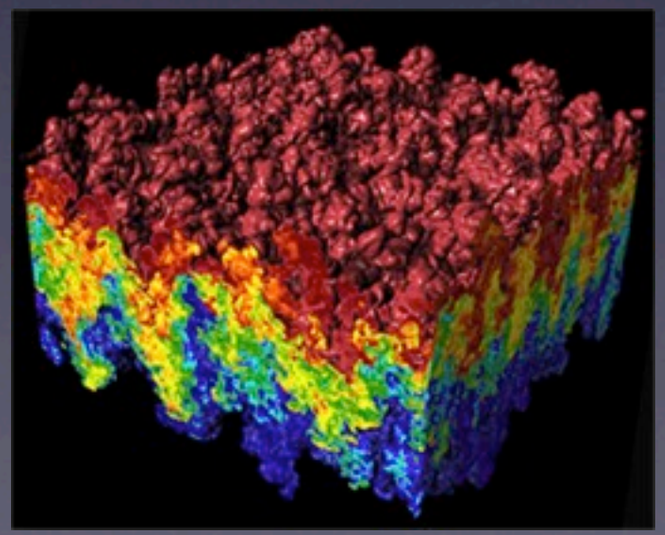
Townsley *et al* (2008)

- Width of mixing layer in self-similar phase,

$$\lambda = \alpha A g t^2$$

(von Neuman & Fermi, 1953)

- Unlike classic Rayleigh-Taylor instability, reactive flame stabilizes surface of flame



Cook & Cabot (2006)

Fire-Polishing Scale

- Timescale for development of Rayleigh-Taylor instability

$$t_{\mathrm{RT}} \sim \sqrt{\frac{\lambda}{Ag}}$$

- Flame-crossing timescale

$$t_{\mathrm{flame}} \sim \lambda / v_{\mathrm{lam}}$$

- Two timescales are comparable at

$$\lambda_{\mathrm{fp}} \simeq \frac{v_{\mathrm{lam}}^2}{Ag}$$

- *On scales < λ_{fp}, action of flame "polishes" the surface, stabilizing it against Rayleigh-Taylor.*

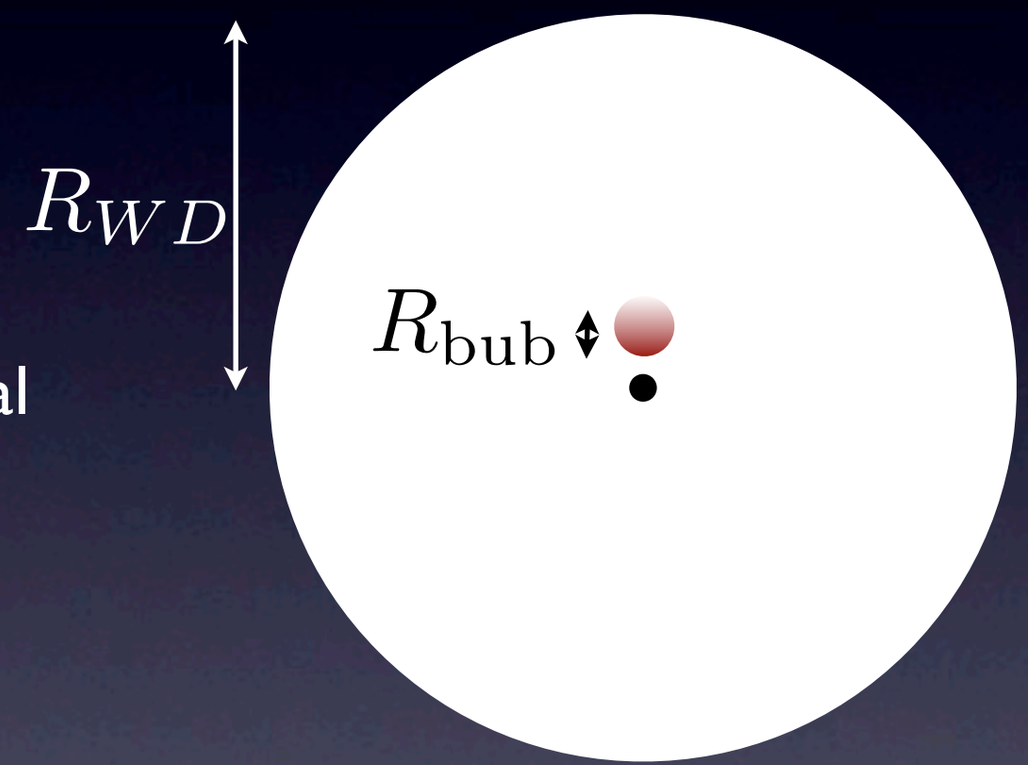
Flame Bubble in an Open Computational Domain for Constant Flame Speed, Gravitational Acceleration, and Changing Density

Flame Speed = 85 km s-1
Gravitational Acceleration = 3 x 109 cm s-2
Initial Density = 2.2 x109 gm cm-3

(1km Detail)

3-D Simulation of a Type Ia Detonation

- Background is a cold white dwarf model in initial equilibrium with initial mass 1.365 Msun with a nuclear equation of state.



- Nuclear bubble is ignited within a spherical region slightly offset from the center of the white dwarf.

- Simulation numerically integrates the fully-coupled Euler, Poisson, and ADR PDEs forward in time.

Fundamental Equations

- Equations solved are Euler equations of hydrodynamics coupled to Poisson's equation for self-gravity and an advection-diffusion reaction model of combustion front :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{v}\rho) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{v}\rho) = -\nabla P - \rho \nabla \Phi$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [\mathbf{v}\,(\rho E + P)] = \rho \mathbf{v} \cdot \nabla \Phi + \rho \epsilon_{\mathrm{nuc}}$$

$$E = U + \frac{1}{2}\rho v^2$$

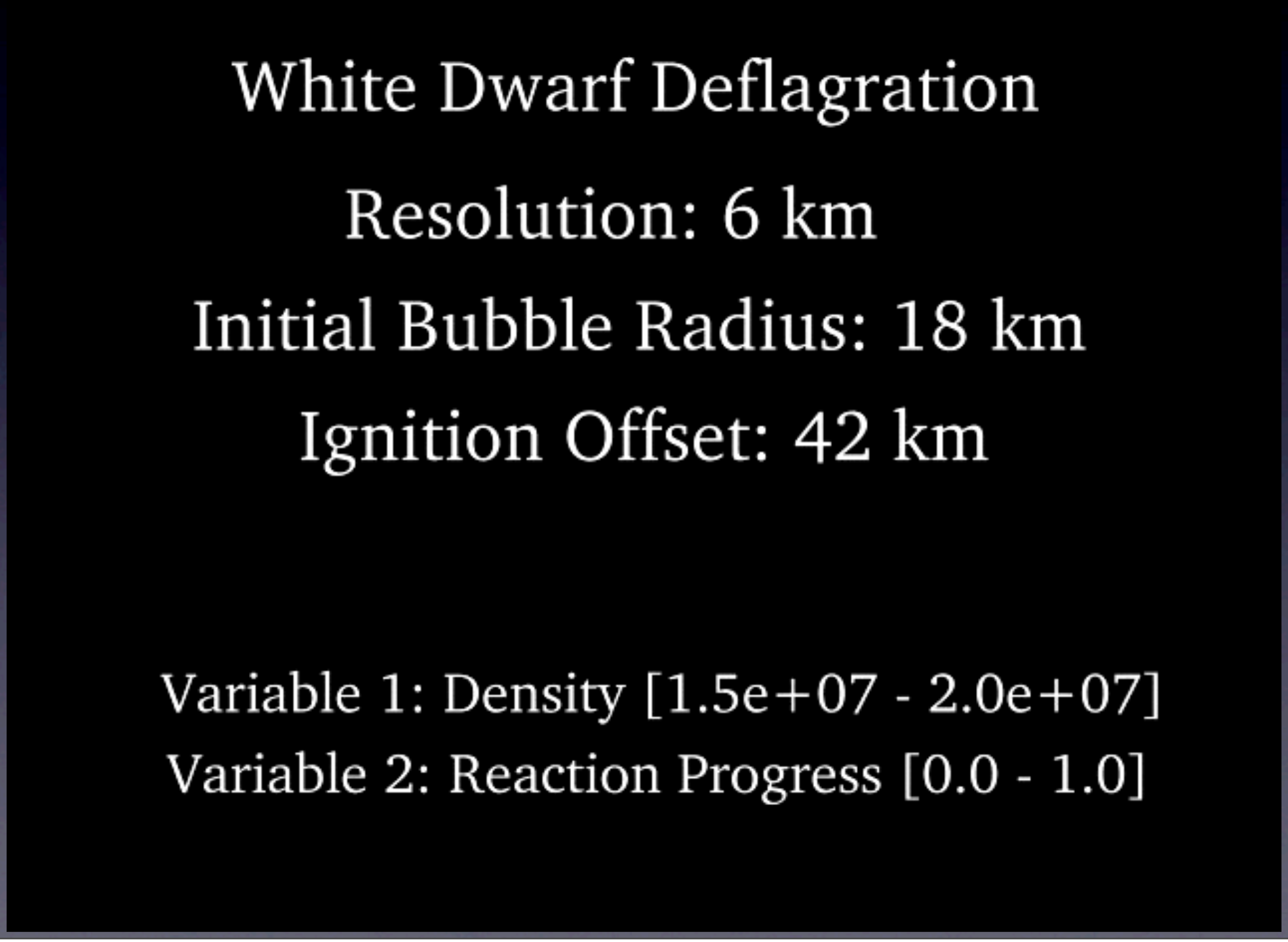
$$\nabla^2 \Phi = 4\pi G \rho$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \kappa \nabla^2 \phi + \frac{1}{\tau} R(\phi)$$

Numerical Methods

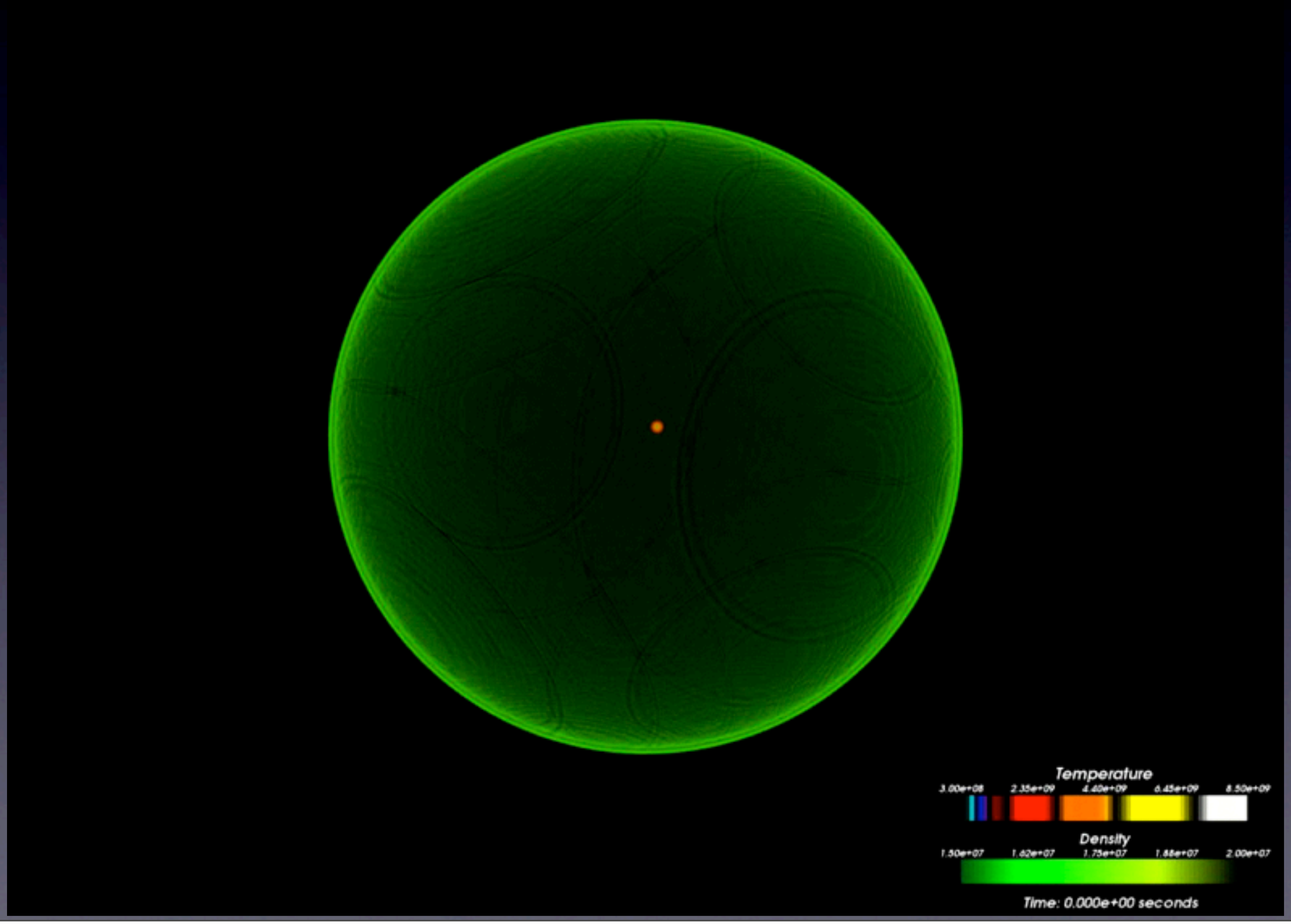
- Equations of hydrodynamics are solved with an explicit, piecewise parabolic method (Colella & Woodward, 1984).
- Self-gravity is solved using a multipole method.
- Source terms for gravity, nuclear burning are operator-split.

Animation of 3D Simulation of Type Ia Supernova GCD Through Deflagration



Jordan et al, 2008

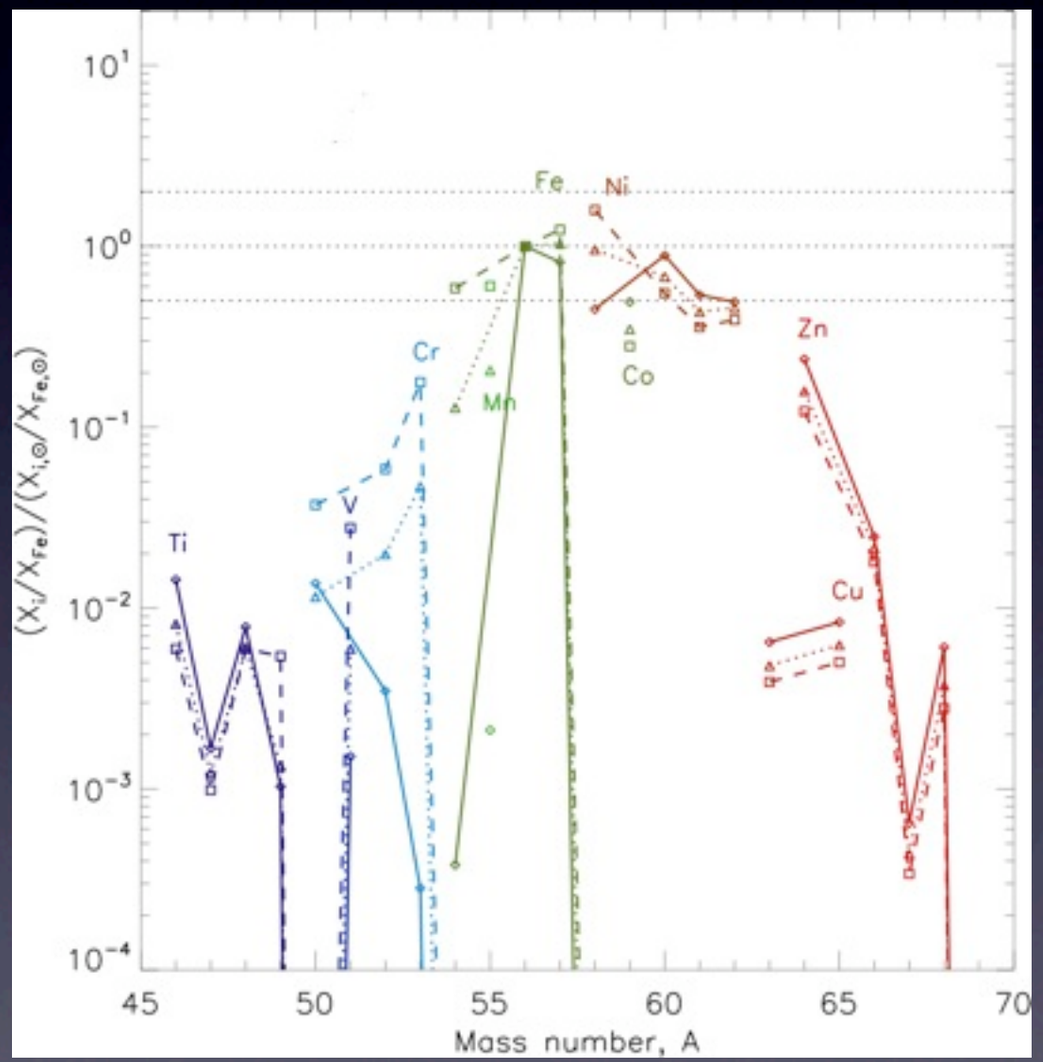
Animation of 3D Simulation of Type Ia Supernova GCD Through Detonation



Jordan *et al*, 2008 Meakin *et al*, 2009

The Problem of Intermediate Mass Elements

- Initial GCD models studied underproduced intermediate mass elements and overproduced Ni.
- Consequently, initial GCD models were generally too luminous in comparison to Branch normal Ia events.
- We have subsequently explored the dependence of the nuclear energy release in the deflagration phase.



Meakin et al, 2009

The Center for Astrophysical Thermonuclear Flashes

Simulation of the Deflagration and Detonation Phases of a Type Ia Supernovae

30 initial bubbles in 100 km radius.
Ignition occurs 80 km from the center of the star.
Hot material is shown in color and stellar surface in green.

This work was supported in part at the University of Chicago by the DOE NNSA ASC ASAP and by the NSF. This work also used computational resources at LBNL NERSC awarded under the INCITE program, which is supported by the DOE Office of Science.

An Advanced Simulation and Computation (ASC)
Academic Strategic Alliances Program (ASAP) Center
at The University of Chicago

ASC

Jordan *et al*, 2012

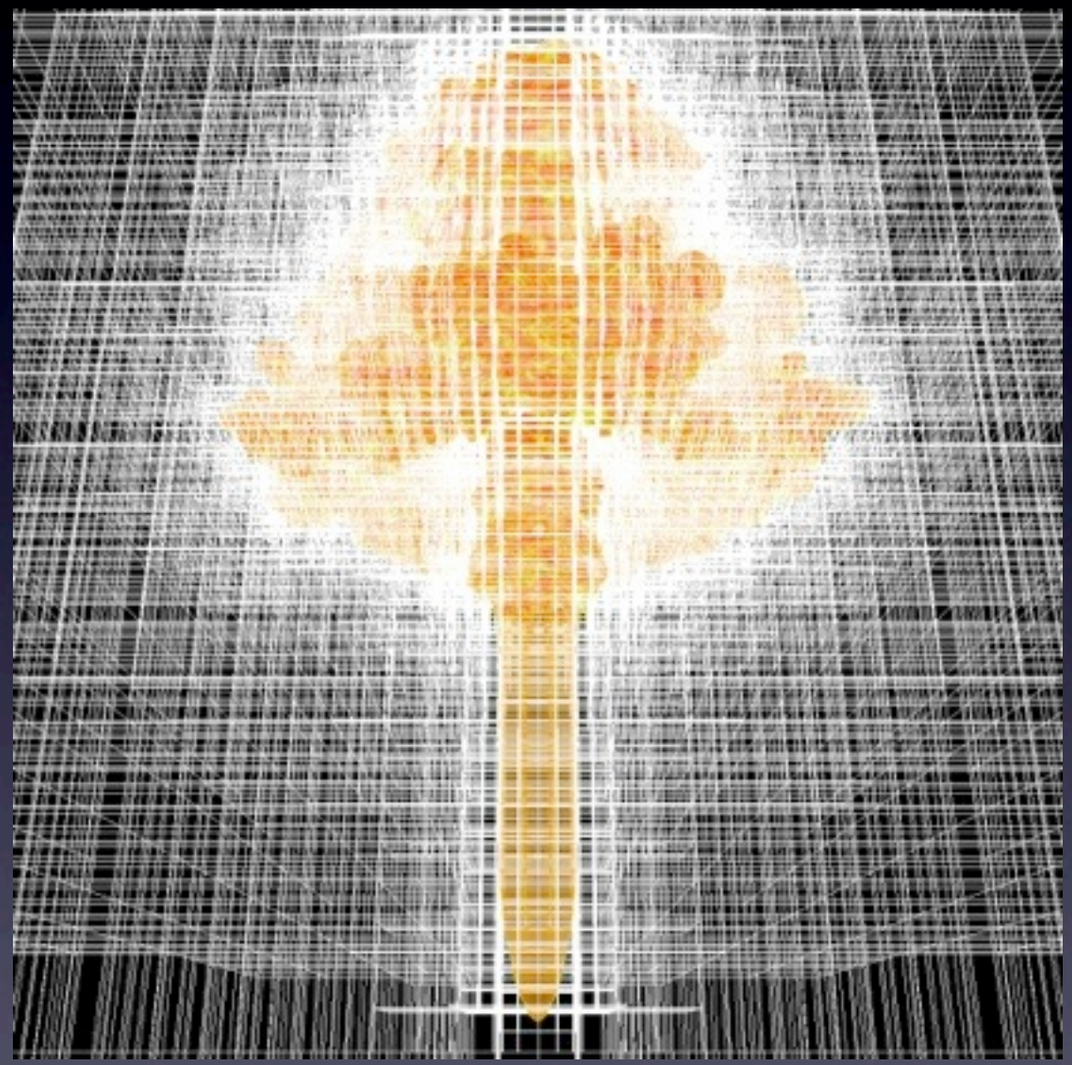
Verification of Flame Models

- Thickness of carbon-burning flame $10^{-5}\mathrm{cm} - 10^{4}\mathrm{cm}$ in the conditions of interest, much smaller than typical grid cell ~ 1 km.
- How does buoyancy-driven combustion in a stratified medium differ from turbulent nuclear burning on an isotropic turbulent background?
- Is it possible to describe the burning rate by a single characteristic turbulent timescale?
- If so, what scale dominates burning - large or small? Integral scale? Fire-polishing scale? Kolmogorov scale?

Adaptive Gridding

- Typical models have finest resolution equivalent to 700 cell zones across the initial diameter of the white dwarf progenitor, 8192 zones across the problem domain.

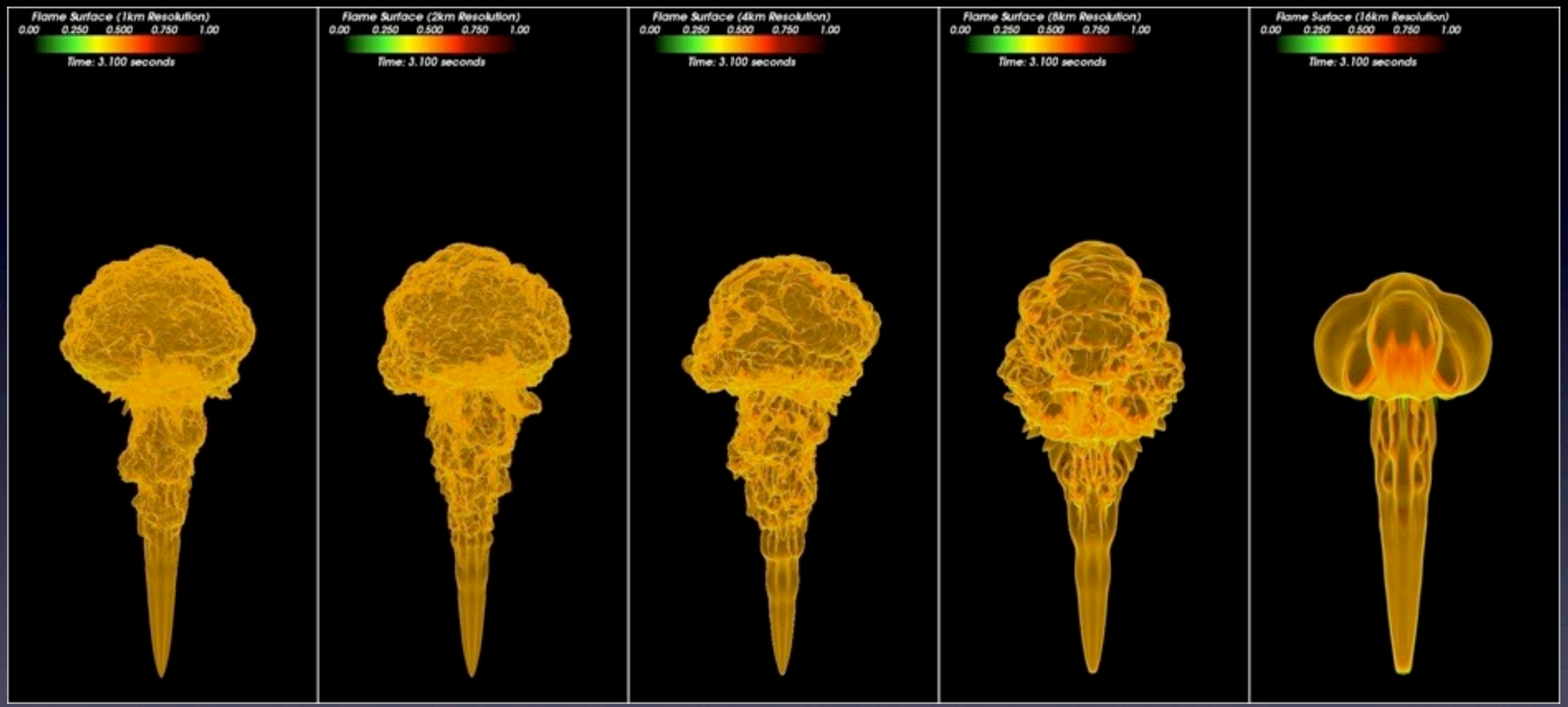
- With adaptive meshing, a single typical simulation followed from ignition to detonation requires roughly 5 10^4 CPU hours on current hardware; about 2 days of wall clock time on 10^3 CPUs.



AMR Grid Refinement of Flame Bubble

Flame Bubble Resolution Study

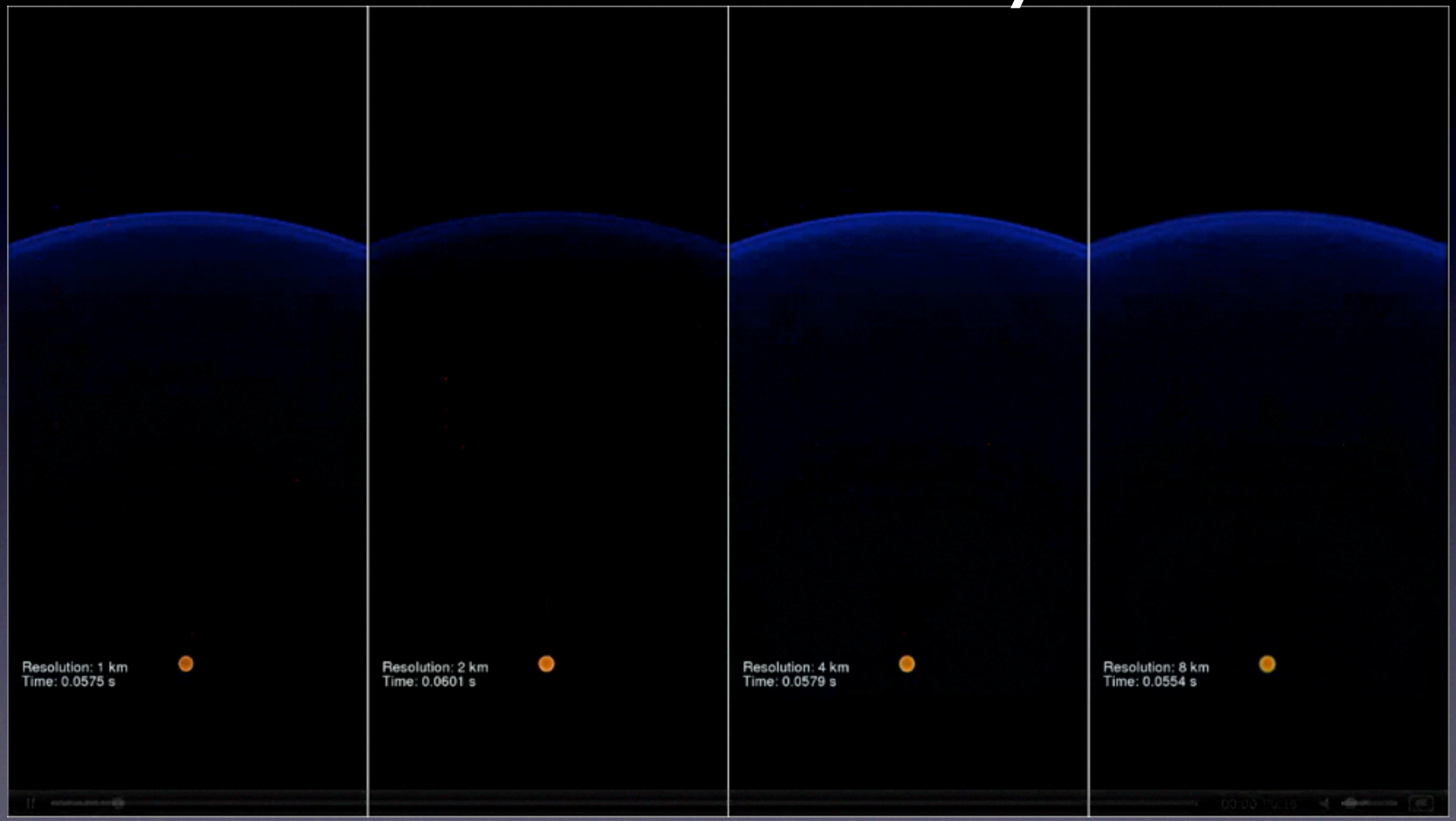
1km 2km 4 km 8 km 16 km



Bair *et al* (2011)

- *Burning area of flame bubble converged for resolutions* $\lesssim \lambda_{\mathrm{fp}}$

Full-Star Deflagration Resolution Study



Single-Degenerate Channel Summary

- Three-dimensional single-degenerate models can now, for the first time, successfully detonate and produce typical brightness SNe Ia.

- Most of the burning in occurs on large scales above fire-polishing scale, and models which resolve this scale are converged.

- Models have predictive capabilities for observational astronomers - in both EM (including optical, UV, and X-ray) as well as gravitational waves

III. The Double-Degenerate Channel

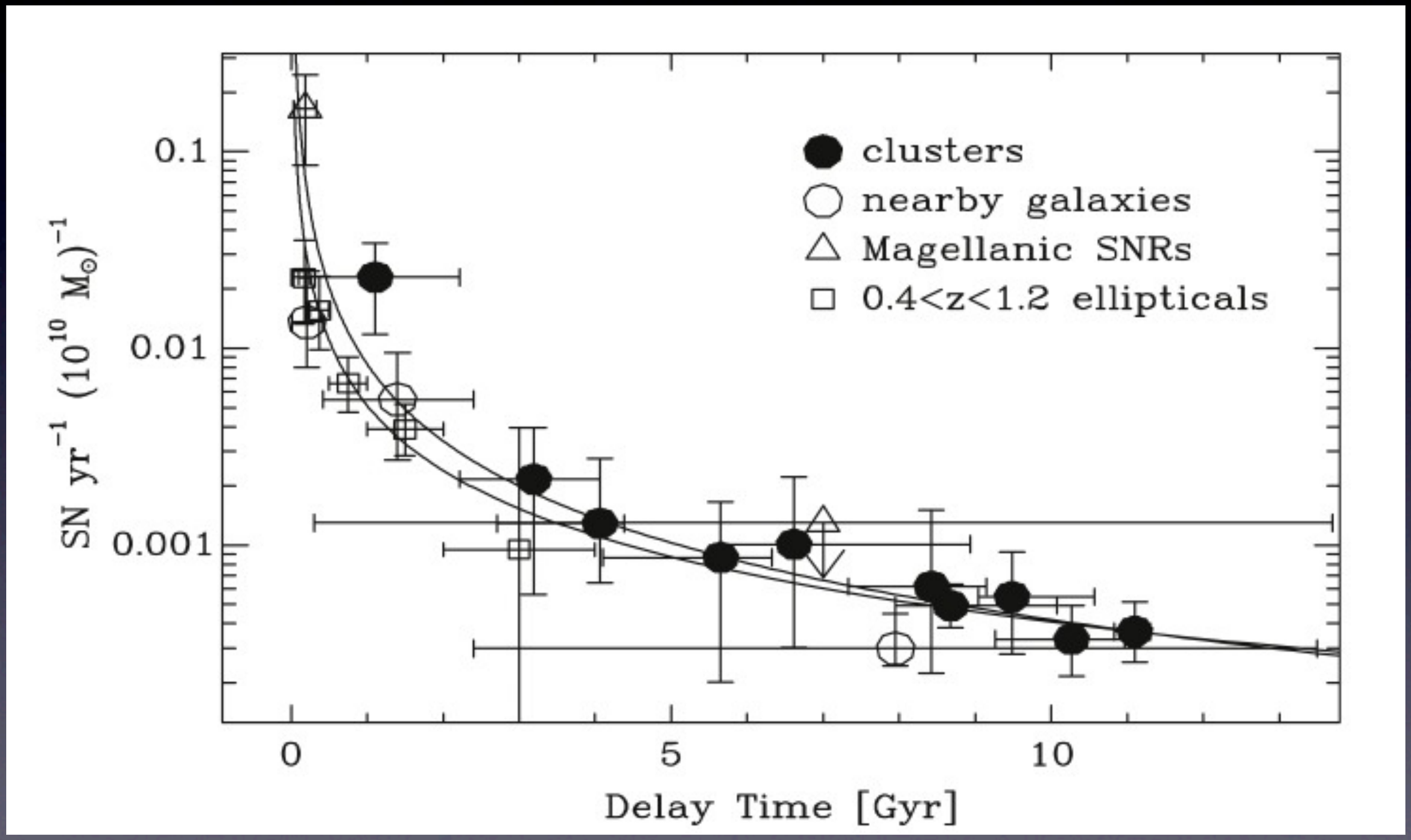
Double-Degenerate Channel

- Recent work based upon delay-time distribution and on 2011fe are consistent with double-degenerates as the origin for some of SNe Ia.

- Multiple observations of DTD show a power law at late times, consistent with DD channel. (Gal-Yam & Maoz, 2004, Totani *et al*, 2008, Badenes *et al*, 2010, Maoz *et al*, 2010)

- Null detections in radio and X-rays in 2011fe constrain mass outflows from SD channel. Moreover, early optical and Swift UV light curves rule out dense circumstellar matter < 10^{10}cm

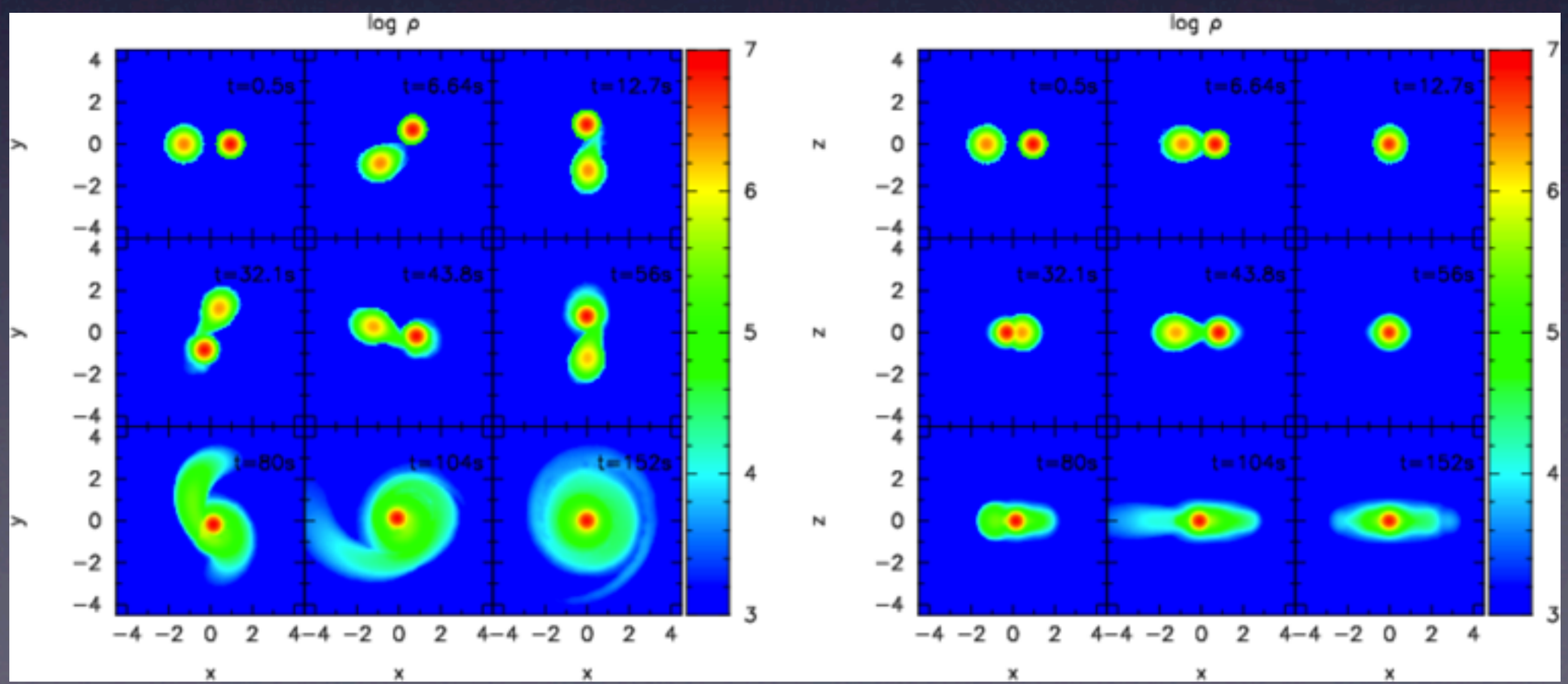
Delay Time Distribution



Maoz et al, 2010

Final Merger of C/O WDs

- Two decades of simulations of final merger of DD channel have finally achieved consensus that mass transfer is unstable. (Mochkovitch & Livio 1989, 1990, Benz *et al*, 1990, Rasio & Shapiro, 1995, Segretain *et al*, 1997, Motl *et al*, 2001, D'Souza *et al*, 2004, Guerrero *et al*, 2004, Loren-Aguilar *et al*, 2009ab, Dan *et al*, 2011)



Loren-Aguilar *et al*, 2009 ab

A Model of Sub-Chandra Channel SNe Ia

- Assuming a Shakura-Sunyaev viscosity, accretion timescale of disk

$$\tau_{\rm acc} = \frac{M_{\rm disk}}{\dot{M}} = \alpha^{-1}\left(\frac{r_{\rm disk}}{h}\right)^2 \tau_{\rm dyn}$$

$$= 2\ {\rm hr}\left(\frac{\alpha}{0.01}\right)^{-1}\left(\frac{r_{\rm disk}/h}{4}\right)\left(\frac{\Omega}{0.2\ s^{-1}}\right)$$

- The heating timescale is then approximately

$$\tau_H = (M_{\rm core}/M_{\rm disk})\tau_{\rm acc} = 5\ \tau_{\rm acc}$$

Or, approximately 10 hours.

Simulating the Sub-Chandra DD Channel

- The sub-Chandra DD channel presents new computational challenges :

Single-Degenerate	Double-Degenerate
Hydrodynamic	Magnetohydrodynamic (or α disk model)
Non-rotating, or Slowly-Rotating WD	Rotating WD Merger + Thick Disk
Evolution over Dynamical Timescale	Evolution over Viscous Timescale

Simulating the Sub-Chandra DD Channel

- Several strategies exist for mitigating these challenges :

Double-Degenerate Physics	Numerical Methods
Magnetohydrodynamics	Unsplit Higher-Order Godunov Solver
Rotating WD Merger + Thick Disk	Corotating Frame
Evolution over Viscous Timescale	Super-Timestepping Higher-Order Schemes Fully-Implicit Methods

Feasibility of Simulating the DD Sub-Chandra Channel SNe Ia

- In general, CPU time for an Eulerian simulation is simply

$$t_{\rm CPU} = N\chi$$

Number of Spacetime Grid Points CPU Time Per Advance

- At a nominal spatial resolution of 1024^2 (~ 100 km), CFL timestep ~ 5 ms, and therefore 10 hr represents 10^7 timesteps.

$$t_{\rm CPU} = 30\ {\rm khr}\left(\frac{N}{1024}\right)^2$$

- For 2-D, this is equivalent to roughly 2 days of wall clock time on a 512-core cluster.

Summary :
Lessons Learned and Open Questions

- 3-D model simulations demonstrate :
  - Successful detonations of Type Ia supernovae in 3D.
  - Convergence when fire-polishing scale is resolved.
- Significant challenges remain :
  - Can we constrain or possibly rule out some models of SD SN Ia by next-generation simulations containing improved initial conditions and models of turbulent nuclear burning?
  - Do sub-Chandra DD merger models yield SNe Ia and not AIC?
  - Can simulated X-ray and UV spectra yield further constraints to the SNe Ia progenitors?

“As long as a branch of science offers an abundance of problems, so long is it alive; a lack of problems foreshadows extinction or the cessation of independent development… It is by the solution of problems that the investigator tests the temper of his steel; he finds new methods and new outlooks, and gains a wider and freer horizon.”

-- David Hilbert, 1900