1. Problem 10.2 from Trefethen & Bau.

2. Problem 11.3 from Trefethen & Bau.

3. Problem 12.2 from Trefethen & Bau.

4. Problem 13.3 from Trefethen & Bau.

5. Prove the following:
   
   (a) \( \kappa(A) \geq 1 \) for any induced matrix norm.
   
   (b) If \( U \) is unitary, then \( \kappa_2(U) = 1, \kappa_2(UA) = \kappa_2(AU) = \kappa_2(A) \).
   
   (c) \( \kappa_2(A) = \sigma_{\text{max}}/\sigma_{\text{min}} \).
   
   (d) If \( A \) is hermitian, then \( \kappa_2(A) = |\lambda|_{\text{max}}/|\lambda|_{\text{min}} \).
   
   (e) If \( Ax = b \) and \((A + \delta A)(x + \delta x) = b\), then \( \|\delta x\|/\|x + \delta x\| \leq \kappa(A) \).

6. Consider \( Ax = b \). Let \( x \) be the exact solution and let \( \tilde{x} \) be an approximate solution. The error is \( e = x - \tilde{x} \) and the residual is \( r = b - A\tilde{x} \).

   (a) Show that \( Ae = r \) and \( \|e\|/\|x\| \leq \kappa(A) \|r\|/\|b\| \).
   
   (b) It follows that if \( A \) is invertible, then \( e = 0 \) if and only if \( r = 0 \), but if \( A \) is ill-conditioned, then the relative error \( \|e\|/\|x\| \) may be large even if the relative residual \( \|r\|/\|b\| \) is small. This occurs in the following example (due to W. Kahan).

   \[
   A = \begin{pmatrix} 1.2969 & 0.8648 \\ 0.2161 & 0.1441 \end{pmatrix}, \quad b = \begin{pmatrix} 0.8642 \\ 0.1440 \end{pmatrix}, \quad x = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \quad \tilde{x}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \tilde{x}_2 = \begin{pmatrix} 0.9911 \\ -0.4870 \end{pmatrix}
   \]

   Show that \( Ax = b \) (using exact arithmetic). Consider \( \tilde{x}_1 \) and \( \tilde{x}_2 \) as approximate solutions and or each one compute the corresponding \( \|e\|_\infty/\|x\|_\infty, \|r\|_\infty/\|b\|_\infty \). Find \( \kappa_\infty(A) \).