Math 552	**Scientific Computing II**	**Spring 2016**
Homework Set 5		Due Friday, 29 April 2016

1. Consider the 2-point BVP

$$\begin{cases} -u'' + u = (\pi^2 \sin \pi x - 2\pi \cos \pi x)e^x \\ u(0) = u(1) = 0\,. \end{cases}$$

The exact solution is $u(x) = e^x \sin \pi x$.

(a) Write a MATLAB script to solve the problem by the FFT method, using the *Discrete Sine Transform* as implemented by *dst.m* applied to the **4th** order centered compact FD scheme, assuming $\sigma \geq 0$ is a **constant**,

$$-\Big(D^2 - \frac{h^2}{12}\sigma D^2\Big)v_i + \sigma v_i = \Big(1 + \frac{h^2}{12}D^2\Big)f_i,$$

where $D^2 = D_+D_-$. Assume a meshsize $h = 1/2^p$, where p is a positive integer. For $p = 1:4$, plot the exact solution ($u(x)$ vs. x) and the numerical solution (v_i vs. x_i), including the boundary points. The 4 plots should appear separately in one figure, with axes labeled and a title for each indicating p. Investigate **subplot** in MATLAB for how to have multiple plots in a single figure window.

(b) For $p = 1:12$ present a table with the following data - column 1: h; column 2: $\|u_h - v_h\|_\infty$; column 3: $\|u_h - v_h\|_\infty / h^4$, where $h = 1/n$. Discuss the trends in each column. Include a copy of your code.

2. Derivative (Neumann) boundary conditions. Consider the model problem

$$-u''(x) + \sigma u(x) = f(x), \quad 0 < x < 1, \quad \sigma \geq 0\,,$$

subject to the *Neumann boundary conditions* $u'(0) = u'(1) = 0$. Find the system of linear equations that results when second-order finite differences are used to discretize this problem at the grid points $x_0, \ldots, x_n$. At the end points, x_0 and x_n, one of many ways to incorporate the boundary using ghost points is to approximate the boundary conditions using a one-sided approximation and take $v_0 = v_1$ and $v_n = v_{n-1}$. How many equations and how many unknowns are there in this problem? Give the matrix that corresponds to this boundary value problem. Is the resulting matrix invertible? Explain.

3. Consider the 2-point BVP with Neumann BCS:

$$\begin{cases} -u'' + u = f \\ u_x(0) = u_x(1) = 0\,. \end{cases}$$

Note that $u(x) = x^2(x-1)^2 e^x$ is the exact solution.

(a) Write a MATLAB script to solve the problem by the FFT method, using the *Discrete Cosine Transform* as implemented by *dct.m* applied to the **2nd** order centered compact FD scheme, assuming $\sigma \geq 0$ is a **constant**,

$$-D^2 v_i + \sigma v_i = f_i,$$

where $D^2 = D_+D_-$. Assume a meshsize $h = 1/2^p$, where p is a positive integer. For $p = 1:4$, plot the exact solution ($u(x)$ vs. x) and the numerical solution (v_i vs. x_i), including the boundary points. The 4 plots should appear separately in one figure, with axes labeled and a title for each indicating p. Investigate **subplot** in MATLAB for how to have multiple plots in a single figure window.

(b) For $p = 1:12$ present a table with the following data - column 1: h; column 2: $\|u_h - v_h\|_\infty$; column 3: $\|u_h - v_h\|_\infty / h^2$, where $h = 1/n$. Discuss the trends in each column. Include a copy of your code.