**MATH 552** **Scientific Computing II** **Spring 2016**

Homework Set 4 Due Friday, 8 April 2016

1. For $N$ even define a discrete grid $x_h$ for $[0, 2\pi]$ by $x_k = k * h$ where $h = 2\pi/N$ and $0 \leq k \leq N-1$. Show
$$e^{i(\frac{N}{2}+j)x_k} = e^{-i(\frac{N}{2}-j)x_k}$$
for $j = 1, 2, \ldots, \frac{N}{2} - 1$. This shows that on such a grid, wave numbers greater than $\frac{N}{2}$ are aliased to wave numbers below $\frac{N}{2}$.

2. Given a real vector $v = (v_1, \ldots, v_{N-1})^T$ the *Discrete Sine Transform* of $v$ is given by $\hat{v} = P^{-1}v$, where $P$ is an $(N-1) \times (N-1)$ matrix with $P_{i,j} = (2/\sqrt{2N}) \sin(ij\pi/N)$ for $i, j = 1, 2, \ldots, N-1$.

   We showed that $\hat{v}$ could be computed using an FFT, implemented by the M-file *dst.m*. For a randomly chosen $v$ and values of $N$ given by
$$N = [64, 96, 128, 256, 368, 512, 1024, 1874, 2048, 3477, 4096]$$
   compute $\hat{v}$ by direct matrix-vector multiplication and also by *dst.m*. Using *tic* and *roc* in MATLAB, plot the cpu time on a *semilogy* plot, and discuss the results. To obtain a reasonably accurate timing, execute each method 500 times and then take the average.

3. Consider the 2-point one-dimensional BVP
$$\begin{cases} -u'' + u = (\pi^2 \sin \pi x - 2\pi \cos \pi x)e^x \\ u(0) = u(1) = 0\,. \end{cases}$$
   The exact solution is $u(x) = e^x \sin(\pi x)$.

   (a) Write a MATLAB script to solve the problem by the FFT method, using the *Discrete Sine Transform* as implemented by *dst.m* applied to the **2nd** order centered FD scheme, assuming $\sigma \geq 0$ is a **constant**,
$$-D^2 v_i + \sigma v_i = f_i,$$
   where $D^2 = D_+ D_-$. Assume a meshsize $h = 1/2^p$, where $p$ is a positive integer. For $p = 1:4$, plot the exact solution ($u(x)$ vs. $x$) and the numerical solution ($v_i$ vs. $x_i$), including the boundary points. The 4 plots should appear separately in one figure, with axes labeled and a title for each indicating $p$. Investigate **subplot** in MATLAB for how to have multiple plots in a single figure window.

   (b) For $p = 1:15$ present a table with the following data - column 1: $h$; column 2: $\|u_h - v_h\|_\infty$; column 3: $\|u_h - v_h\|_\infty / h^2$, where $h = 1/n$. Discuss the trends in each column. Include a copy of your code.